EXPANSION MODES OF FOREIGN SUBSIDIARIES BY MULTINATIONAL CORPORATIONS

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Abstract

Multinational corporations expand foreign subsidiaries by switching resources from other locations or by scaling up these subsidiaries independently. Although the two options are alternatives, the choice between them and their interactions with each other have not been investigated. This study develops a model that casts these options as alternatives and provides several novel insights. These results help explain paradoxical findings in international strategy research and motivate future empirical studies on the means by which firms expand their foreign subsidiaries.

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INTRODUCTION

Foreign subsidiaries of multinational corporations (MNCs) are often located in uncertain and dynamically changing countries. With shifts in local product markets, labor markets, and local institutional environment, as well as in exchange rates for the local currency, executives must choose how to expand foreign subsidiaries (Kogut, 1983, 1989; Trigeorgis, 1996). The developing real options perspective on such decisions emphasizes that MNCs can access upside opportunities and limit downside losses by altering commitments to foreign subsidiaries with the arrival of new information on host country environments. The real options perspective therefore provides distinctive insights into international diversification that can inform research on the management and consequences of multinationality (Hennart, 2011; Oh and Contractor, 2012).

Research on real options identifies two ways in which MNCs expand in host countries. First, an MNC can use a *switching option* by withdrawing resources from a domestic subsidiary and redeploying them to a foreign subsidiary (Chang and Matsumoto, 2021; Kogut and Kulatilaka, 1994; Huchzermeier and Cohen, 1996). Such withdrawal involves "non-scale free" resources (e.g., capital, physical assets, labor, and input procurement), whose use in one subsidiary entails an opportunity cost of not switching them to a better performing subsidiary (Levinthal and Wu, 2010). This view portrays the MNC as a dynamic network whose flexibility depends on the dispersion of assets across countries with unique environments (Belderbos et al., 2014, 2019, 2020; Belderbos and Zou, 2007; Sundaram and Black, 1992). Second, the MNC can use a within-country *growth option* by buying resources from factor markets in the host country and scaling up its foreign subsidiary on a standalone basis (Chang and Rosenzweig, 2001; Chi and McGuire, 1996; Chi and Seth, 2009).

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Although the two ways for an MNC to expand its foreign subsidiaries were sometimes regarded as a portfolio of options (Kogut, 1989; Trigeorgis, 1996), more often they were studied separately from each other. Existing research has relied on conceptual framing, formal valuation models, and empirical analyses to study antecedents and consequences of each option while implicitly assuming away another option. For instance, the literature on multinationality and the switching option has built on early conceptual models (Kogut, 1983; Kogut and Kulatilaka, 1994; Trigeorgis, 1996) to test empirically whether multinationality provides useful operational flexibility and performance or risk advantages (Allen and Pantzalis, 1996; Belderbos, Tong and Wu, 2014; Reuer and Leiblein, 2000; Tong and Reuer, 2007). That research also began to isolate conditions under which the option to switch operations across countries is created (Fisch and Zschoche, 2012) and used (Rangan, 1998). Just as these studies focused on the switching option and ignored the growth option within the host country, other research focused on the option to expand through staged investment without considering the switching option in a firm's portfolio (Kogut, 1991; Folta, 1998; Chi, 2000; Folta and Miller, 2002; Cuypers and Martin, 2010; Tong and Li, 2013; Smit et al., 2017). Chi et al. (2019) attributed the limited account of the interplay between the two alternative modes of foreign expansion to the complexity of portfolios of growth and switching options and to the analytical challenges of modeling them.

This study uses a formal model to restore the portfolio approach and to do a comparative analysis of switching and growth options in MNCs. The idea that the two options are alternatives to each other is an input to, rather than an output of, the model. In turn, the key output is a set of predictions regarding the use of each expansion option in the presence of another option. Such research could inform executives whether they should engage in switching operations across countries or in scaling up a foreign subsidiary in a particular situation. More broadly, the model

with both options responds to calls to study interactions between real options (Anand et al., 2007; Folta and O'Brien, 2004; Vassolo et al., 2004), including a recent call to formally model the specific interaction between the switching option and the growth option in the portfolio held by an MNC (Chi et al., 2019). Although the formal model in this study is based on the methods used in previous formal valuation models (Chi, 2000, Huchzermeier and Cohen, 1996; Kogut and Kulatilaka, 1994), the model extends this work by deriving the best possible use of the expansion options over time. Also, the model reliably identifies the causal relationships in the context with two options that are complex individually and interact with each other.

The model incorporates the following determinants of the two expansion options identified in previous research: (a) the current returns in an MNC's subsidiaries, (b) the cost of exercising the switching option (hereafter, for brevity, "the switching cost"), (c) the cost of exercising the growth option (hereafter, "the growth cost"), and (d) uncertainty of future returns in an MNC's subsidiaries. Some results (e.g., that the use of the switching option is reduced by the switching cost and enhanced by uncertainty, or that the use of the growth option is reduced by the growth cost), are intuitive and validate the model. Meanwhile, there are eleven unique and unintuitive results. While they all are discussed in detail below, four groups of results stand out.

First, an MNC was believed to expand its foreign subsidiary via the switching option when that subsidiary strongly outperformed its other subsidiary, and via the growth option when the foreign subsidiary performed well in absolute terms. The model indicates that these intuitive conditions are not sufficient for the expansion. The intuition breaks down because the direct effect of performance of the foreign subsidiary, alone (for the growth option) or relative to the domestic subsidiary (for the switching option), is confounded by three factors: (i) the optimal time for the use of the option may not have come yet, so even the strongest performance of the

foreign subsidiary, in absolute terms or relative to another subsidiary, cannot induce expansion; (ii) the optimal time for the use of the option under the conditions of the strongest performance of the foreign subsidiary may have already passed, and such performance does not lead to further expansion; and (iii) another expansion option may currently be more profitable, so the focal option is not used despite the performance conditions that seem ideal for the focal option alone.

Second, it is indeed intuitive that the switching cost negatively affects the use of the switching option to expand an MNC's subsidiary, and that the growth cost has a strong negative effect on the use of the growth option to expand an MNC's subsidiary. The model indicates that this intuition is right, but incomplete, because it is based on the neglect of the interactions between the costs of implementing the expansion option and uncertainty. This study shows that the effects of both switching costs and growth costs are negatively moderated by uncertainty.

Third, uncertainty was considered a key facilitator for the use of the expansion options. Again, this popular idea neglected the interactions between uncertainty and other determinants of the expansion options. While the model confirms that uncertainty monotonically raises the use of the switching option, the switching cost positively moderates the effect of uncertainty such that low switching costs may even make that effect trivial. For the growth option, the interaction between uncertainty and the cost of implementing that option is so strong that it can even flip the positive effect of uncertainty on the use of the option to ultimately become a negative effect. Notably, the odds that the firm will use the growth option decline in uncertainty if the growth cost is low and rise in uncertainty only if the cost is high.

Finally, it is intuitive that the use of an expansion option is affected negatively by the cost of using that option and positively by the cost of using another option. The model indicates that these ideas require elaboration because they neglect the interactions between the options.

Namely, the cost of implementing one option positively moderates how strongly the use of another option is suppressed by its own cost. Overall, the results of the model provide strong reasons to consider portfolios of growth and switching options and their interplay.

By addressing Chi et al.'s (2019) call for models with both the switching and the growth options, this study aims to improve the ability of empirical research to characterize how MNCs expand foreign subsidiaries. For example, that an MNCs can expand its foreign subsidiary through the growth option can explain why the switching option was not fully used by MNCs despite strong advantages of foreign subsidiaries over other subsidiaries in those MNCs (Rangan, 1998). Likewise, the consideration of determinants of the expansion options other than performance of subsidiaries (i.e., of the costs of implementing these options and of uncertainty) warrants more comprehensive empirical investigation and can begin to explain empirical findings that appeared counterintuitive otherwise. Furthermore, given the important role that uncertainty played in verbal accounts and in exploratory models of the use of the expansion options, the elaboration of the non-monotonic effect of uncertainty on the expansion of foreign subsidiaries by MNCs lays the groundwork for more accurate specification of future empirical models. Finally, the results that are derived from the formal model introduced below can also serve as a starting point for the development of heuristics, or rules of thumb, for executives of MNCs who manage alternative expansion modes for their foreign subsidiaries.

MODEL

The model involves a firm that has a foreign subsidiary in addition to its domestic business. Specifically, immediately before the initial time $t = 0$ in the model (i.e., at time $t = 0 - \partial t$), the firm used half (i.e., proportion $m_{i0-\partial t} = 0.5$) of resources in foreign business i; the firm used another half (i.e., proportion $(1 - m_{i0-\partial t}) = 0.5$) of resources in domestic business j. This initial

configuration is denoted as $M_{0-\partial t}=1$. At any time t before the end of the lifecycle of its resources $t=T$, the firm can double its participation in the foreign business by using one of two real options. First, the firm can use the switching option: it can switch all resources that were initially used in business j to business i, which is denoted as $M_t=2$. Second, the firm can use the growth option: it can buy additional resources for business i, which is denoted as $M_t=3$. If the firm stays in its initial configuration at time t, this situation is denoted as $M_t=1$. The model consists of three parts: (1) a specification of returns in the firm's businesses, (2) a specification of the two expansion options, and (3) a description of how the firm uses those options. Items "1" and "2" are described in turn below, whereas item "3" is developed in ONLINE APPENDIX.

Returns in the firm's businesses

Returns in the firm's businesses are uncertain. In particular, the margin C_{it} in the foreign business and the margin C_{jt} in the domestic business follow geometric Brownian motions:

$$C_{it}=C_{i0}e^{\left[\left(\mu_i-\frac{\sigma_i^2}{2}\right)t+\sigma_i W_{it}\right]} \qquad (1)$$

$$C_{jt}=C_{j0}e^{\left[\left(\mu_j-\frac{\sigma_j^2}{2}\right)t+\sigma_j W_{jt}\right]} \qquad (2)$$

$$dW_{it}\,dW_{jt}=\rho\,dt\,. \qquad (3)$$

In Equations 1–3, C_{i0} and C_{j0} are margins in businesses i and j at the initial time $t=0$; μ_i and μ_j are drifts for the margins; σ_i and σ_j are volatilities of the margins that capture uncertainty; and W_{it} and W_{jt} are Brownian motions with correlation ρ. This specification, which is prevalent in modeling real options, makes a reasonable assumption that the uncertain margins become more difficult to predict the farther the margins are projected into the future.

Real options for expanding the foreign business

Switching option. The first option with which the firm can expand its foreign business is to switch its resources to that business. If resources are switched to the foreign business i, the net margin that is earned with the resources that are withdrawn from the domestic business j is lower than the regular margin C_{it} in the foreign business i, by the marginal switching cost s. Like in Sakhartov and Folta (2015), the full switching cost S_t^x, is a product of (a) the marginal switching cost s of a unit of resources; (b) the amount $(1-m_{i0-\partial t})=0.5$ of resources that are switched to i; and (c) the current realization C_{it}^x of margin C_{it} in the recipient business:

$$S_t^x = s(1-m_{i0-\partial t})C_{it}^x = 0.5sC_{it}^x. \qquad (4)$$

Equation 4 leads to the following statement of the expected net present value V_t^{xS} of the firm that uses the switching option to expand its foreign subsidiary:

$$V_t^{xS} = \begin{cases} -S_t^x + C_{it}^x + e^{-r\partial t}E^{P^i}\left[V_{t+\partial t}^{xS} \mid \left(M_t^* = 2, x\right)\right] & \text{if } M_{t-\partial t} = 1 \\ C_{it}^x + e^{-r\partial t}E^{P^i}\left[V_{t+\partial t}^{xS} \mid \left(M_t^* = 2, x\right)\right] & \text{if } M_{t-\partial t} = 2 \end{cases}. \qquad (5)$$

In Equation 5, $E^{P^i}\left[V_{t+\partial t}^{xS} \mid \left(M_t^* = 2, x\right)\right]$ is the expectation with respect to the probability distribution P^i for C_{it}, r is a risk-free interest rate, and $V_{t+\partial t}^{xS}$ is the net present value in the immediate next time $(t+\partial t)$. Expectation $E^{P^i}[\cdot]$ is conditioned on the current or the past choice to use the switching option $(M_t^* = 2)$. This expectation is assessed when margin C_{it} is in state x. While the first line in Equation 5 corresponds to the situation where resources were not switched before time t (i.e., $M_{t-\partial t} = 1$); the second line is for the situation where resources were switched to i before time t (i.e., $M_{t-\partial t} = 2$) and, thus, no switching cost is incurred at time t.

Growth option. The second way in which the firm can expand its foreign subsidiary is to use the growth option and to buy additional resources for that unit. If the firm does so, it pays the price G_t equal to the discounted net present value that similar firms would accumulate, on average, in deploying the amount $m_{i0-\partial t}$ of resources in the foreign business i from time $\tau = t$ to time $\tau = T$. The price is increased by the premium $\gamma \geq 0$, which the firm pays to buy extra resources on the factor market. This premium characterizes the incurred growth cost. Formally,

$$G_t = (1+\gamma) m_{i0-\partial t} \sum_{\tau=t}^{T} e^{-r(\tau-t)} \hat{C}_{i\tau} = 0.5(1+\gamma) \sum_{\tau=t}^{T} e^{-r(\tau-t)} \hat{C}_{i\tau}, \qquad (6)$$

where $\hat{C}_{i\tau}$ is the average margin that is earned by firms that operate in business i at time τ. In addition to the interpretation that the growth cost is a transaction cost in the imperfect factor market, the growth cost can be seen as the cost of overcoming 'barriers to entry' (Bain, 2013). One factor that makes such barriers critical in the context of foreign expansion is the 'liability of foreignness' (Zaheer, 1995). Although the firm has already invested in the country, it can still encounter such a liability as it expands in the country. Also, if for competitive reasons the expansion should be executed quickly, the MNC incurs 'time-compression diseconomies' (Jiang, et al., 2014), which represent an additional ramification of the growth cost. Thus, parameter γ captures these or other sources of the cost of exercising the growth option.

With this specification of the growth cost, the expected net present value V_t^{xyG} of the firm when it uses the growth option to expand its foreign business can be expressed as follows:

$$V_t^{xyG} = \begin{cases} -G_t + C_{it}^x + 0.5C_{jt}^y + e^{-r\partial t} E^{Pij}\left[V_{t+\partial t}^{xyG} \mid \left(M_t^* = 3, x, y\right)\right] & \text{if } M_{t-\partial t} = 1 \\ C_{it}^x + 0.5C_{jt}^y + e^{-r\partial t} E^{Pij}\left[V_{t+\partial t}^{xyG} \mid \left(M_t^* = 3, x, y\right)\right] & \text{if } M_{t-\partial t} = 3 \end{cases}. \qquad (7)$$

Here, $V_{t+\partial t}^{xyG}$ is the net present value at time $(t+\partial t)$. Expectation $E^{P^{ij}}\left[V_{t+\partial t}^{xyG} \mid M_t^* = (3,x,y)\right]$ is taken with respect to the joint probability distribution P^{ij} for C_{it} and C_{jt}. This expectation is conditioned on the current or the past choice to use the growth option $(M_t^* = 3)$ and is estimated when margins C_{it} and C_{jt} are in states x and y. Whereas the first line in Equation 7 considers the case where the growth option is exercised exactly at time t (i.e., $M_{t-\partial t} = 1$), the second line is relevant when the growth option was exercised before time t (i.e., $M_{t-\partial t} = 3$).

The switching and the growth options are not obligations for the MNC. They are used only if doing so makes the firm better off. A natural alternative for the firm at any time is to continue without expanding its foreign subsidiary. As is common in the real options modeling, the use of the two expansion options is characterized based on the principle of dynamic optimality (Bellman, 1957). Also, as is typical of the evaluation of complex options, the solution is semi-analytical, and its completion involves an established numerical approach of Boyle, Evnine and Gibbs (1989). Details of the solution are provided in ONLINE APPENDIX.

RESULTS

Results of the model are split into four groups. The first group specifies the use of the expansion options in terms of current returns in the firm's markets. The second group focuses on how the switching cost and uncertainty affect the use of the two options. The third group considers how the growth cost and uncertainty determine the use of the options. The fourth group investigates how the switching cost and the growth cost together influence the use of the two options. If the variation of a parameter is not required in a considered result, the following values are used: $s = 10$, $\gamma = 0.015$, $C_{i0} = C_{j0} = 0.08$, $\sigma = \sigma_i = \sigma_j = 0.2$, $\rho = 0$, $T = 1$, and $r = 0.08$. If the variation of a parameter is needed, the range for the parameter is reported with the result.

Implications of current returns for the use of the expansion options

The evolution of margins C_{it} and C_{jt} is contained within a rectangular pyramid with the apex at time $t = 0$. Figure 1 dissects that pyramid at four points in time. In each of the four panels, two sectional views represent 'snapshots' for the probabilities for the use of the two options over realizations C_{it}^{x} and C_{jt}^{y} at time t.

Insert Figure 1 here

Panel A of Figure 1 illustrates the start of the use of the switching option. As the red-colored area of the left-hand side plot demonstrates, that use begins in the top left corner of the plot. In that corner, the margin in the foreign market C_{i9} has its highest possible realization $C_{i9}^{\max}$ and the margin in the domestic market C_{j9} has its lowest realization $C_{j9}^{\min}$ in the considered time. Although the co-occurrence of $C_{i9}^{\max}$ and $C_{j9}^{\min}$ is rather unlikely, if the firm happens to be in that state, it will switch resources because difference $\left(C_{i9}^{\max} - C_{j9}^{\min}\right)$ represents a high opportunity cost to remaining in the underperforming business. While this scenario with the strongest advantage $\left(C_{it}^{\max} - C_{jt}^{\min}\right)$ of the foreign market over the domestic market occurred in the top left corners of the 'snapshots' for all preceding time steps, step 9 is the first time when this condition favorable for the exercise of the switching option entails such exercise. This fact reflects a rational delay, or inertia, in the use of the switching option that, despite the strongest advantage of the destination over the origin, is fully aligned with dynamic optimality. Hence, predicting the use of that option demands the *joint* analysis of performance and time. In other words, the strong advantage of the destination market for switching resources in the MNC over the original market

is necessary but not sufficient for the exercising of such switching. Two additional explanations for the non-sufficiency of performance are developed below.

Panel B shows the start of the use of the growth option. The use of the growth option begins in the top right corner where the margin in the foreign market C_{i19} has its highest possible realization $C_{i19}^{\max}$ in the respective time, thus making the foreign market most attractive for the expansion through the growth option. Although the same scenario with the highest return $C_{it}^{\max}$ in the foreign market happened in the top right corners of the 'snapshots' for all preceding time steps, step 19 is the first time when this condition favorable for the growth option plays out, thus indicating a delay in the use of the growth option despite the strongest attractiveness of the foreign market. Accordingly, predicting the use of the growth option also requires the *joint* analysis of performance and time: the high return in the foreign market is necessary but not sufficient for the exercising of the growth option. The rational delay in the use of the growth option reflects the first manifestation of non-sufficiency of performance.

The second manifestation of non-sufficiency of the highest return in the foreign market for predicting the use of the growth option is that such use critically depends on returns in the domestic market. That use occurs only if the margin in the domestic market is close to its highest value $C_{j19}^{\max}$ because, with lower returns in the domestic market, the firm can use the switching option. Thus, the second source of the non-sufficiency of the highest return in the subsidiary for predicting the use of the growth option is that this highest return may activate the switching option instead of the growth option. Multiple states that invite such switching form a straight line with a slope greater than 45 degrees, on which the condition $C_{i19}^{x} > C_{j19}^{y}$ justifies the switching from j to i. Below that line, the use of the switching is unprofitable. Why does the dark-blue

area to the north-west of that line show no use of the switching even when $C^{x}_{i19} > C^{y}_{j19}$, specifically in the top left corner where the difference $\left(C^{\max}_{i19} - C^{\min}_{j19}\right)$ asumes the highest value? Also, why does that area show no use of the growth option even with $C^{\max}_{i19}$? This lack of use of the expansion options reveals the third manifestation of non-sufficiency of performance: the combinations of the two margins in the considered area are formed by evolving from their states $C^{\max}_{i9}$ and $C^{\min}_{j9}$ with which the switching was already used earlier, as shown in Panel A.

Panel C of Figure 1 indicates the evolution of the use of the two options. The line for the switching option continues to expand along with the extension of possible states for returns over time and maintains the same slope of greater than 45 degrees so that $C^{x}_{i100} > C^{y}_{j100}$. In turn, states that lead to the use of the growth option combine into a horizontal line in the respective plot in Panel C. That horizontal line is longer than in Panel B and is distant from the top margin of the plot. Why does the dark-blue rectangular area above the horizontal line reveal no use of the growth option if higher values of C^{x}_{i100} in that area make the growth option more attractive? This happens because the high values of C^{x}_{i100} in that rectangular area have evolved from their state $C^{\max}_{i19}$ with which the growth option was already exercised, as shown in Panel B. It can also be seen that the position of the left end of the horizontal line for the use of the growth option coincides with the position of the right end of the sloped line for the use of the switching option. This result indicates how the choice between the options maps onto the margin in the domestic business: with lower values of that margin, the firm prefers to use the switching option; whereas, with higher values of that margin, the firm chooses the growth option.

Panel D of Figure 1 reports further evolution in the use of the two options. Like in Panel C, the location of the left end of the horizontal line for the use of the growth option matches the

location of the right end of the line with the slope of more than 45 degrees for the use of the switching option. Furthermore, the horizontal line for the use of the growth option becomes even more distant from the top margin of the plot than in Panel B, thus getting closer to the middle of the plot in the vertical dimension. That middle represents the natural threshold for the attractiveness of the growth option because it captures the basis for the price that the firm would have to pay in buying additional resources for the growth. Below that line, that form of expanding the foreign business is unprofitable for the firm.

Figure 2 extends Panel D of Figure 1 with two additional 'snapshots' where one option is disallowed by the model. These plots demonstrate the conditions, in terms of returns in the two businesses that are intrinsic to each option and not artefacts of the presence of another option; these plots are also instrumental for understanding what interrupts the line for each option in plots where both options are present. Thus, in the absence of the growth option, the line for the switching option proceeds uninterrupted to the top margin of the respective plot while maintaining the slope of greater than 45 degrees so that $C^{x}_{i151} > C^{y}_{j151}$. The area below that line represents combinations of the margins with which the use of the switching option is unprofitable. In the area above that line, the firm also refrains from exercising the switching option in the considered time, but for a different reason. As discussed above, the area above the line represents cases where the switching option was already exercised in the past more profitably based on dynamic optimality (Bellman, 1957). In turn, when the switching option is absent, the horizontal line for the growth option proceeds down to the lowest possible value for the margin in the domestic business. The area below that line involves values of the margin in the foreign business with which the use of the growth option is unprofitable. In the area above

that line, the firm avoids using the switching option in the considered time because the growth option was already used in the past more-profitably based on dynamic optimality.

Insert Figure 2 here

To sum up, although the MNC expands its foreign subsidiary via the switching option when that subsidiary most strongly outperforms the MNC's domestic subsidiary and via the growth option when the foreign subsidiary has the highest performance in absolute terms; these ideas are true only when an observer picks the right time for the observation (i.e., step 9 for the switching option and step 19 for the growth option). Otherwise, the effect of performance is confounded by three factors: (i) the optimal time for the option may not have come yet; (ii) the optimal time for the option may have already passed; and (iii) another option may be more profitable. The unique results in Figure 1 are as follows: (a) there is a delay in the use of the switching option, even when the foreign subsidiary most strongly outperforms the firm's other subsidiary; (b) there is a delay in the use of the growth option, even when the foreign subsidiary has the highest performance in absolute terms; (c) only right after such optimal delay, the firm uses the switching option if and only if the foreign market most strongly outperforms the domestic market; (d) only right after such optimal delay, the firm uses the growth option if and only if the foreign market has the highest performance; (e) as the firm's resources age, the use of the switching option demands that the foreign business outperform the domestic business by a margin *and that the foreign business do not perform too well in absolute terms*; (f) as the firm's resources age, the use of the growth option demands that the foreign business outperform the average for its performance by a margin *and that the domestic business do not perform too poorly in absolute terms*. The absence of another option in Figure 2 relaxes the confounding item "iii" and the italicized parts of results "e" and "f" above.

Implications of the switching cost and uncertainty for the use of the expansion options

Figure 3 demonstrates how the switching cost s and uncertainty σ jointly affect the cumulative odds that the firm will have expanded its foreign operations by the middle of the lifecycle of its resources (i.e., by time step 100 out of 200). Specifically, Figure 3 predicts the following three outcomes: (a) the probability of using the switching option when the growth option is also allowed (i.e., Panel A), (b) the probability of using the growth option when the switching option is also allowed (i.e., Panel B), and (c) the probability of using the switching option when the growth option is disallowed (i.e., Panel C). To reduce the dimensionality of results and avoid multi-way interactions (i.e., among σ_i, σ_j, and s), in Figure 3 volatilities of the margin in the foreign market and in the domestic market are the same $(\sigma = \sigma_i = \sigma_j)$.

Insert Figure 3 here

Panel A of Figure 3 changes its tone from red to dark blue in the direction from the top left corner to the bottom right corner. Accordingly, when both expansion options are available, the use of the switching option declines in the switching cost and increases in uncertainty. The negative effect of the switching cost is intuitive: the net payoff to the switching option is reduced by the total switching cost S_t^x, which in turn depends directly on the marginal switching cost s. The positive effect of uncertainty on the probability of switching is less intuitive. An explanation that needs to be ruled out is that the positive effect of uncertainty on the probability of switching is an artifact of the presence of the growth option that, in turn, is influenced by uncertainty. This explanation is indeed rejected in Panel C, where the growth option is disallowed.

To scrutinize how uncertainty and the switching cost affect the use of the switching option, two additional plots are created based on the left-side figure in Panel C. In particular, the middle plot in Panel C dissects the contour map in that panel at the bottom and the top margins.

As expected, slopes of both lines are negative but differ from each other. With low uncertainty, the change in the switching cost from zero to 50 entails the drop of 0.45 in the odds that the MNC uses the switching option; with high uncertainty, the respective drop is only 0.07. This difference reveals that the negative effect of the switching cost on the probability of using the switching option is strongly negatively moderated by uncertainty. This interaction was implied in Kogut and Kulatilaka (1994) and is better understood based on the right-side plot in Panel C.

The right-side plot in Panel C dissects the contour map in that panel at the left and at the right margins. Both lines in that plot have positive slopes, thus confirming that uncertainty increases the use of the switching option. What makes the use of the switching option grow in uncertainty? This monotonic positive effect stems from the condition necessary for the use of the switching option: the foreign business should outperform the domestic business by, at least, the switching cost. Because uncertainty expands the bounds for the vacillation of returns in both markets and enhances the vacillation of those returns between upper and lower bounds, high uncertainty enables the arrival of that necessary condition. In other words, with higher uncertainty and thus with broader and more intense vacillation of returns in the two markets, returns in the foreign subsidiary are more likely to become above returns in the domestic subsidiary at least once over the lifecycle of the MNC's resources. By contrast, when uncertainty is lower, returns in the two markets vacillate less intensely, and the return in the MNC's foreign subsidiary is less likely to exceed the return in the domestic subsidiary.

In addition to diagnosing the direct effect of uncertainty, the right-side plot in Panel C reconfirms the interaction between uncertainty and the switching cost and is useful in explaining this interaction. The positive slopes for the blue and red lines markedly differ from each other. With the low switching cost, the rise in uncertainty from 0.1 to 1.1 increases the odds that the

MNC uses the switching option by only 0.0044; with the high switching cost, the respective rise is 0.38. The difference shows that the effect of uncertainty on the use of the switching option is positively moderated by the switching cost. When the switching cost is trivial, even a tiny advantage in the foreign market over the domestic market induces the switching from the latter to the former, and scenarios where the switching option is used abound. In that case, the switching is very likely in general and the blue line has a consistently high altitude over the base of the panel; uncertainty can increase that inherently high propensity to switch only by a thin margin. Conversely, the high switching cost makes the MNC more inert in using the switching option; the firm switches only when the advantage in the foreign market exceeds this high switching cost. With low uncertainty at the left margin of the panel, the bands for the vacillation of the margins in the two markets are narrow and such scenarios favorable to the switching are unlikely. By contrast, high uncertainty extends the bands for the two margins, thus spurring the scenarios that are favorable to the switching. As a result, the right end of the red line rises far above the left end, thus demonstrating that uncertainty matters much more when the switching cost is high than when the switching cost is low.

Regarding the effects of the switching cost and uncertainty on the use of the growth option, Panel B of Figure 3 changes its tone from dark blue in the top left corner to red in the bottom right corner. This change reveals that, when both expansion options are present, the odds of using the growth option increase in the switching cost and decline in uncertainty. The positive effect of the switching cost on the use of the growth option holds because higher costs make the use of the switching option less profitable, thus making the firm prefer the growth option. Explaining the negative effect of uncertainty on the use of the growth option needs to rule out the

chance that this effect derives from the presence of the switching option that, in turn, is affected by uncertainty. This issue is addressed in the next section.

To conclude this section, the unique results in Figure 3 are as follows: (a) uncertainty increases the use of the switching option, (b) the switching cost positively moderates the effect of uncertainty on the use of the switching, and (c) uncertainty reduces the use of the growth option (this idea is revisited below).

Implications of the growth cost and uncertainty for the use of the expansion options

Figure 4 explores how the growth cost γ and uncertainty σ jointly affect the cumulative odds that the firm will have expanded its foreign operations by the middle of the lifecycle of its resources. In particular, Figure 4 considers the following three outcomes: (a) the probability of using the growth option when the switching option is present (i.e., Panel A), (b) the probability of using the switching option when the growth option is allowed (i.e., Panel B), and (c) the probability of using the growth option when the switching option is absent (i.e., Panel C).

Insert Figure 4 here

Panel A of Figure 4 demonstrates that, when both options are available, the growth cost reduces the use of the growth option. This negative effect is intuitive: the net payoff to the growth option is reduced by the price G_t that the firm pays for the expansion and that, in turn, includes the premium γ. By contrast, uncertainty has a complex effect on the use of the growth option. Specifically, with low growth costs, uncertainty reduces the use of the growth option; whereas, with high growth costs, uncertainty propels the exercise of the growth option. This complex effect contrasts with the positive effect of uncertainty on the odds of switching. An explanation that needs to be ruled out is that this complex effect derives from the presence of the switching option. This explanation is ruled out in Panel C, where the switching option is absent.

To scrutinize how uncertainty and the growth cost affect the use of the growth option, the middle plot in Panel C dissects the contour map in that panel at the bottom and the top margins. Although the slopes of both lines are negative, they are very differently so. With low uncertainty, the rise in the growth cost from zero to 0.15 entails the drop of 0.46 in the odds of using the growth option; with high uncertainty, the respective drop is only 0.08. Thus, uncertainty negatively moderates the negative effect of the growth cost on the use of the growth option. Although this moderation is akin to the interaction between uncertainty and the switching cost, its unique ramifications are further considered in the right-side plot of Panel C in Figure 4.

The right-side plot in Panel C dissects the contour map in that panel at the left and the right margins. Whereas the slope of the red line is positive, the slope of the blue line is negative. Notably, with the high growth cost, the rise in uncertainty from 0.1 to 1.1 leads to the increase of 0.24 in the use of the growth option; with the low growth cost, such odds drop by 0.15. This change in the sign of the effect suggests that the interaction between uncertainty and the growth cost, which was observed in the middle plot in Panel C, is so strong that it reverses the ultimate effect of uncertainty when the growth cost is low.

What makes uncertainty stimulate the use of the growth option when the growth cost is high (like with the switching option) and suppress the use of the growth option when the growth cost is low (in contrast to the switching option)? This complex effect derives from the exercise condition unique to the growth option: the foreign business should outperform its own average by, at least, the growth cost. When the growth cost is high, the foreign business should outperform its own average by a lot to justify the use of the growth option. This condition places the threshold for the exercising the growth option above most of possible paths for returns in the foreign business, especially when such paths are contained in amplitude by low uncertainty. In

this case, a high amplitude of the vacillation of returns in the foreign business due to high uncertainty is required for the threshold to be hit and the option to be exercised. In other words, high uncertainty enables the use of the growth option despite high growth costs. When the growth cost is trivial, the threshold value of returns in the foreign business reflects the mean for such returns. Lower uncertainty makes it easier for returns in the foreign business to hit that threshold because lower uncertainty contains the vacillation of those returns in the closer proximity to their mean. By contrast, with high uncertainty, many possible paths for future returns diverge from the mean by a lot; and, if this divergence is below the mean, such paths never hit the threshold for the exercising of the growth option.[1] Thus, when the growth option is unencumbered by high costs, uncertainty reduces the odds that the option will ever be exercised, explaining why the right end of the blue line gets below the left end in right side plot in Panel C.

With regard to the effects of the growth cost and of uncertainty on the use of the switching option, Panel B of Figure 4 changes its color from red in the top right corner to dark blue in the bottom left corner. Accordingly, when both options are present, the odds that the firm will use the switching option increase in the growth cost and continue to increase in uncertainty. The positive effect of the growth cost on the use of the switching option occurs because higher growth costs make the use of the growth option less profitable, thus making the firm choose the switching option. The positive effect of uncertainty was explained in the previous section.

To sum up the analysis of the implications of the growth cost and uncertainty, the novel result in Figure 4 are as follows: uncertainty reduces the use of the growth option when the

[1] Given that it is very easy for a random variable with multiple draws taken over time to get above its own mean at least once anyway, the most critical question in this case becomes how many paths never hit the threshold rather than how many do. With strong divergence of paths for returns in the foreign market (i.e., sequences of draws for the respective random variable) from the mean provided by high uncertainty, relatively more paths go deeper down and never get the growth option into the money. The fact that a path may also strongly diverge upward due to higher uncertainty does not seem to be as consequential because a single occasion of the path crossing the mean suffices for the option exercise anyway; so, the path going further up does not change the instance of exercising the option on that path after it first hits the threshold.

growth cost is low and increases that use when the growth cost is high. As the central plot in Panel C indicates, this result can also be reformulated as a negative moderation of the negative effect of the growth cost on the use of the growth option.

Implications of the switching cost and the growth cost for the use of the expansion options

Figure 5 depicts how the switching cost s and the growth cost γ jointly affect the cumulative odds that the firm will have expanded its foreign subsidiary by the middle of the lifecycle of its resources. Figure 5 predicts the following outcomes: (a) the probability of using the switching option when the growth option is absent (i.e., Panel A), (b) the probability of using the growth option when the switching option is absent (i.e., Panel B), (c) the probability of using the switching option when the both options are present (i.e., Panel C), and (d) the probability of using the growth option when both options are present (i.e., Panel D).

Insert Figure 5 here

When the MNC can expand its foreign subsidiary only through the switching option in Panel A, the growth cost does not affect the use of the switching option, whereas the switching cost promotes such use. Likewise, when the MNC can use only the growth option in Panel B, the switching cost does not influence such use, whereas the growth cost raises such use. Thus, both panels emulate the design of studies of foreign expansion by MNCs that have assumed that each option can be investigated empirically in isolation from another expansion option.

By contrast with Panel A, Panels C shows that, in a more inclusive view where the MNC also has the growth option, the growth cost does influence the use of the switching option. This influence is manifested in two ways. First, the change of color along the vertical dimension in Panel C reveals that the MNC is more likely to use the switching option when the growth cost is higher. This effect reveals the substitution between the two differently costly options. Second,

although the switching cost still reduces the use of the switching option, this effect becomes weaker when the growth cost is lower. Thus, the growth cost positively moderates the negative effect of the switching cost on the use of the switching option. As a result of this positive moderation, the ignorance of the growth option would overinflate the negative effect of the switching cost on the use of the switching option in most of Panel A. Another way to map this bias is that, only when the growth cost is prohibitively high at the top of Panel C, will the magnitude of the negative effect of the switching cost on the use of the switching option be as high as in Panel A where the growth option is ignored. The positive moderation in Panel C can be explained as follows. When the growth option is prohibitively expensive, the MNC does not regard it as a viable expansion option, and the choice is between exercising the switching option or not expanding at all. Thus, the use of the switching option for the expansion of the MNC's foreign subsidiary depends mostly on the switching cost. By contrast, when the growth cost is lower, the MNC chooses among the switching option, the growth option, and not expanding the foreign subsidiary. Hence, the effect of the switching cost on the use of the switching option is attenuated by the possible use of the less costly growth option.

Panels D displays another more inclusive scenario where the MNC does not ignore the switching option. By contrast with Panel B, in Panel D the switching cost does affect the use of the growth option. As a manifestation of a direct effect that indicates the substitution between the two differently costly options, the alteration of color along the horizontal dimension in Panel D reveals that the switching cost enhances the use of the growth option. As a manifestation of a moderation effect, when the switching cost is low at the left margin of Panel D, the negative impact of the growth cost on the use of the growth option is lower. Thus, the negative effect of the growth cost on the use of the growth option is reinforced by the switching cost. An important

implication of this moderation is that the neglect of the switching option exaggerates the negative effect of the growth cost on the use of the growth option in most of Panel B. That is, only when the switching cost is prohibitively high at the right margin of Panel D will the magnitude of the negative effect of the growth cost on the use of the growth option be as high as in Panel B where the switching option is disregarded. The revealed moderation has the following explanation. When the switching option is prohibitively expensive, the MNC does not count on it, and the choice is between using the growth option or not expanding at all. Thus, the use of the growth option for the expansion of the MNC's foreign subsidiary depends mostly on the growth cost. By contrast, when the switching cost is lower, the MNC chooses among the growth option, the switching option, and not expanding the foreign subsidiary. Accordingly, the effect of the growth cost on the use of the growth option is muted by the possible use of the switching option.

The unique results in Figure 5 can be summarized as follows: (a) the growth cost positively moderates the negative effect of the switching cost on the use of the switching option, and (b) the switching cost positively moderates the negative effect of the growth cost on the use of the growth option.

Summary of theoretical predictions on the expansion of foreign subsidiaries in MNCs

The theoretical results presented above can be summarized as the following hypotheses.

Hypothesis 1. Even when the foreign subsidiary most strongly outperforms another subsidiary in the firm, there is a delay in the use of the switching option.

Hypothesis 2. Even when the foreign subsidiary has the highest performance, there is a delay in the use of the growth option.

Hypothesis 3. As the firm's resources age, the exercise of the switching option demands not only that the foreign business outperform the domestic business but also that the foreign business do not have high performance.

Hypothesis 4. As the firm's resources age further, the exercise of the growth option demands not only that the foreign business performs above average but also that the domestic business do not have poor performance.

Hypothesis 5. Uncertainty positively affects the use of the switching option.

Hypothesis 6. The switching cost positively moderates the positive effect of uncertainty on the use of the switching option.

Hypothesis 7. Uncertainty negatively moderates the negative effect of the growth cost on the use of the growth option.

Hypothesis 8. The growth cost positively moderates the negative effect of the switching cost on the use of the switching option.

Hypothesis 9. The switching cost positively moderates the negative effect of the growth cost on the use of the growth option.

DISCUSSION

MNCs confront high uncertainty in foreign countries where they locate subsidiaries. Strong fluctuations in product and labor markets, foreign exchange rates, and economic and institutional contexts of these countries allow MNCs to expand their foreign subsidiaries. Notably, an MNC can switch capital, physical assets, labor, and input procurement to a foreign subsidiary from other countries (Chang and Matsumoto, 2021; Kogut and Kulatilaka, 1994; Huchzermeier and Cohen, 1996); or it can scale up a foreign subsidiary by buying resources necessary for the expansion from factor markets in the host country (Chang and Rosenzweig, 2001; Chi and

McGuire, 1996; Chi and Seth, 2009; Tong et al., 2008). The two ways to expand represent a portfolio of options (Chi et al., 2019) and should be studied accordingly (Martin et al., 2007). However, these options have often been studied separately from each other. This study shows that this can lead to inaccurate theoretical predictions.

For empirical research, this implies that models that examine the use of switching options may conflate what appears to be "do nothing" with the use of growth options, just as models examining the use of growth options may inadvertently treat the use of switching options as "do nothing." The effects of uncertainty, expansion costs, and other determinants of option exercise often involve interactions that have been underappreciated. It is therefore important to appreciate interactions among multiple options firms have to expand foreign businesses (Trigeorgis, 1996).

The contributions of this study are based on a formal model that casts the two expansion modes as a portfolio of options the MNC has. This approach responds to the calls to investigate interactions between various real options, including the recent specific call regarding the interplay between the switching and the growth options in an MNC's portfolio (Chi et al., 2019). The model derives several unique insights that can inform future research and practice. First, neither the strongest advantage of the foreign subsidiary over other subsidiaries in the MNC suffices to justify the switching of the MNC's resources to that subsidiary, nor does the strongest performance of that subsidiary in absolute terms justify its expansion through the growth option. Each of these options is exercised in the specific time that reflects what is optimal for the MNC in the long run. Besides, the presence of another expansion option confounds the use of the focal option based on the performance criteria appropriate for its standalone exercising. Second, although the cost of implementing each expansion option suppresses the use of that option, this effect depends on uncertainty. This study derives multiple interactions, with which the intuitive

negative effect of the cost of implementing an expansion option on the use of that option can decline, and even become trivial. Third, while uncertainty monotonically raises the odds that the MNC expands its foreign subsidiary through the switching option, such uncertainty can increase the use of the growth option only when the growth cost is high. Otherwise, the use of the growth option declines in uncertainty. Finally, the cost of implementing one expansion option reinforces the negative effect of the cost of another expansion option on the use of that another option.

By considering both expansion options, these results can improve the ability of empirical research to interpret the expansion of foreign subsidiaries by MNCs. As one example, it is no longer surprising and does not have to be interpreted as a deviation from the real options theory that an MNC refrains from switching its resources to a foreign subsidiary that substantially outperforms other units in that MNC (Rangan, 1998). The real options can explain such refraining with the preference of the MNC to expand its foreign subsidiary through the growth option or with determinants of the expansion options other than performance of the foreign subsidiary. Ultimately, the theoretical insights that are developed in this study can also have normative implications: they can provide the basis for heuristics to be used in MNCs.

Extensions to this research can proceed in several directions. For example, the two modes of expansion of foreign subsidiaries in MNCs can have implications for firm boundaries and transaction costs (Trigeorgis and Reuer, 2017). It would be interesting to assess how transaction cost considerations influence conditions with which the two expansion modes are selected, given that various uncertainties in host countries affect boundary of the firm choices per transaction cost theory as well as optimal resource allocation choices per real options theory. Other factors not considered here, such as financial constraints, the operation of internal capital markets, and institutional considerations in factor markets may come into play too. It would also be valuable

to examine how firms' initial motives to enter countries, such as market seeking or resource seeking, affect how firms expand foreign subsidiaries. Like previous research on the use of real options by MNCs and on the performance implications of such options, this study takes the existence of the MNC and the extension options as given (Belderbos and Zou, 2007; Belderbos et al., 2014; Chung et al., 2010; Dasu and Li, 1997; Kogut and Kulatilaka, 1994; Rangan, 1989; Reuer and Leiblein, 2000; Tong and Reuer, 2007). For instance, the degree to which a multinational firm seeks to satisfy a local market, such as by following a multidomestic strategy, may inhibit the exercise of switching options. More generally, conditions prevailing upon market entry (e.g., motives, entry modes, localization needs and policies, *etc*.) can have implications for subsequent growth, and real options research might also investigate the potential impact of factors surrounding the decisions to purchase and exercise options. The model might also be elaborated to consider the implications of firms' local or more centralized capabilities to seize upon the switching and growth options. Finally, it would be interesting to evaluate formally how the two expansion options, separately and jointly, affect corporate risk. Whereas this theoretical work has recently started for the switching option (e.g., Sakhartov, 2022), similar assessment can be done for the growth option and for the combination of the two options. Such derivations can help empirical researcher structure analyses based on formal predictions and further extend empirical research on this topic (e.g., Belderbos et al., 2014; Ioulianou et al., 2021; Reuer and Leiblein 2000). Research in directions such as these would help future scholarship address option portfolios and interactions using the comparative approach developed here for firm decision-making, and such study could also generate valuable heuristics for managers facing alternative modes of growth.

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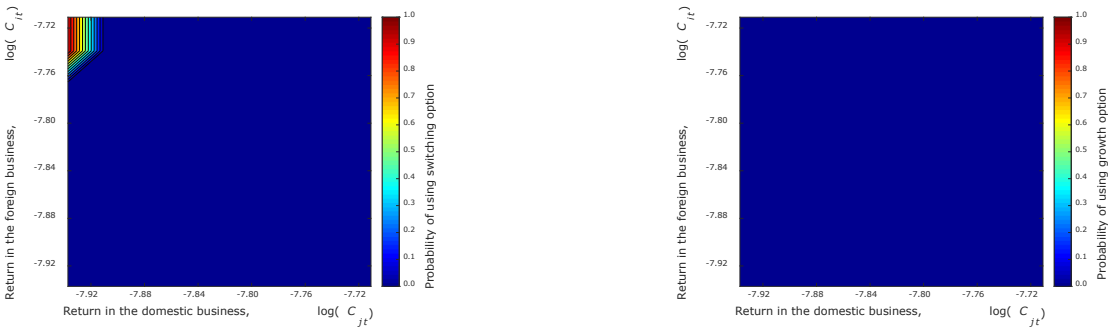

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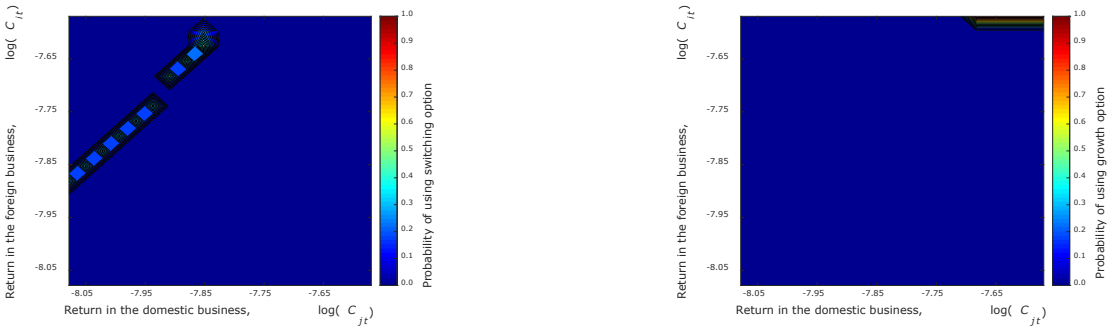

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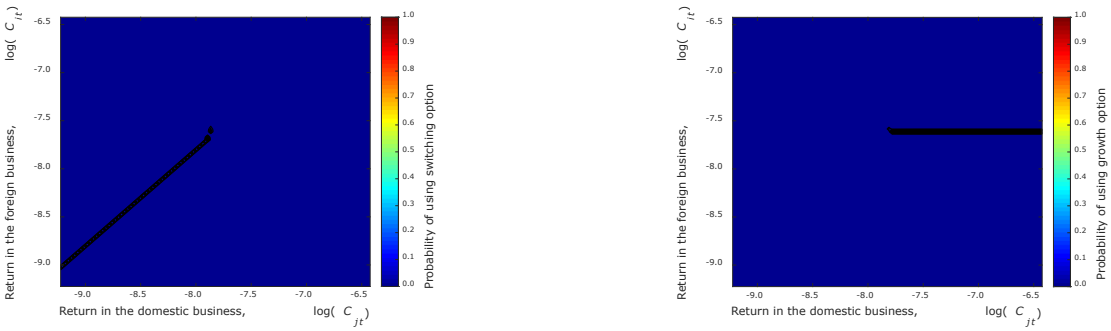

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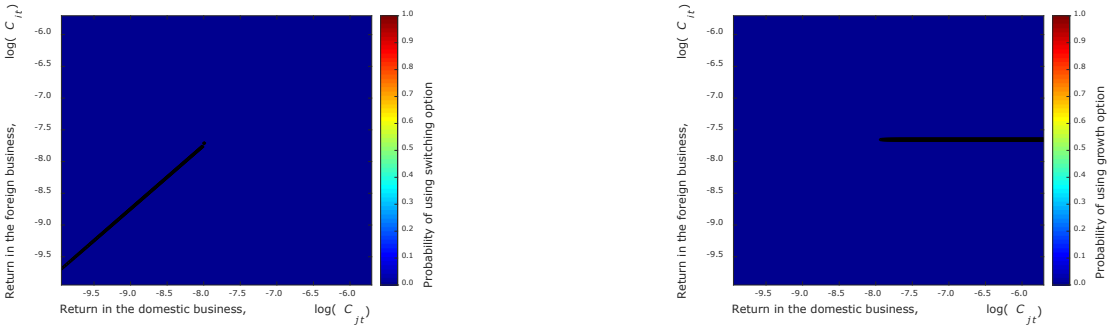
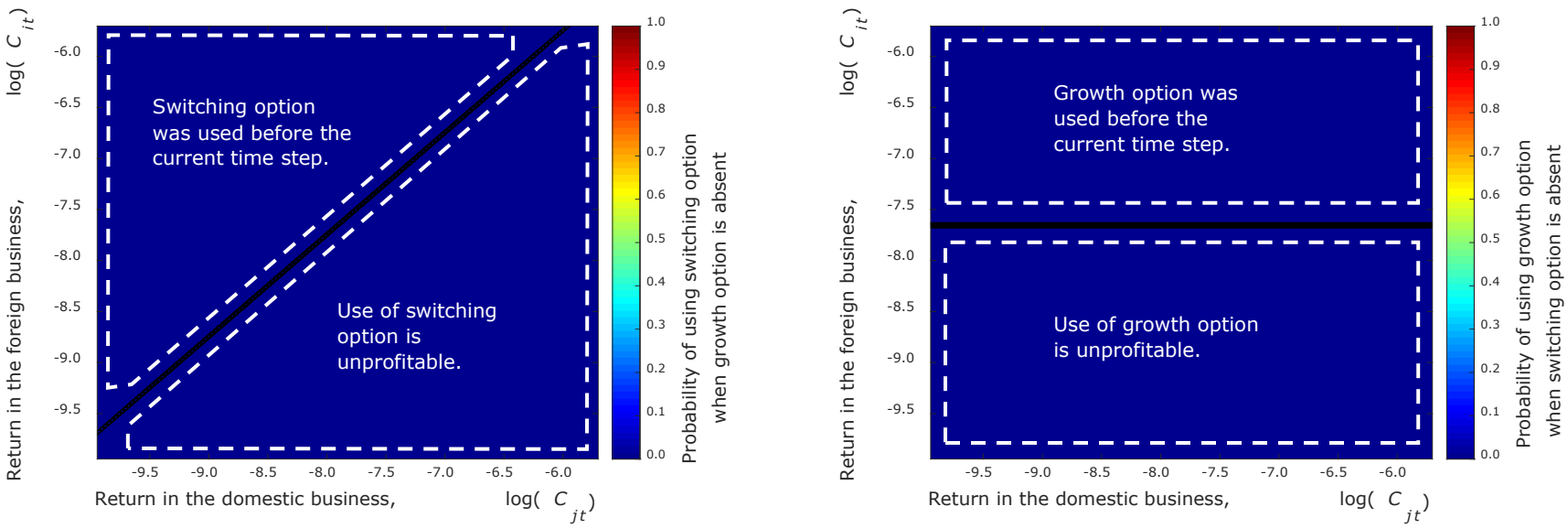


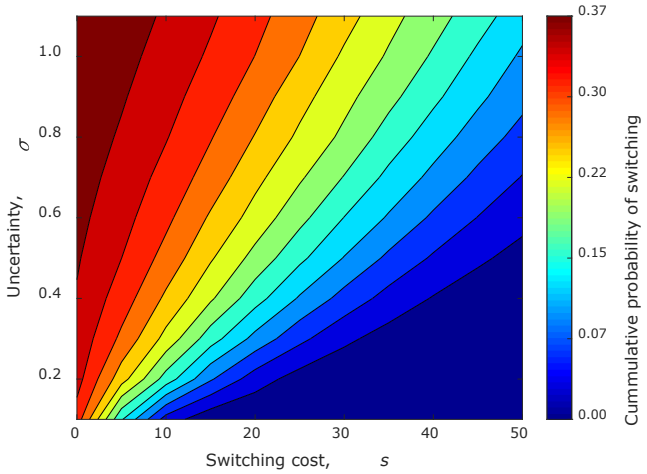
D. Time step 151 out of 200

FIGURE 1. Use of the switching option and of the growth option under different realizations of uncertain returns in the domestic and the foreign businesses (both options are present)

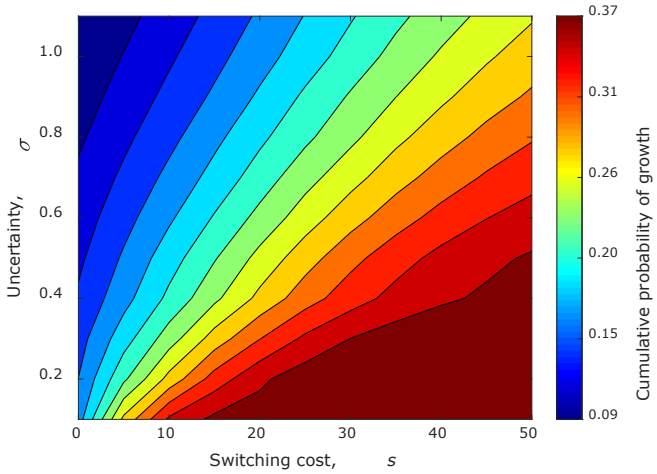


Time step 151 out of 200

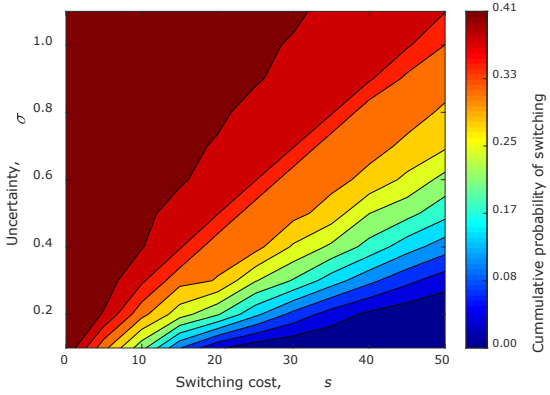
FIGURE 2. Use of the switching option and of the growth option under different realizations of uncertain returns in the domestic and the foreign businesses (only one option is present)

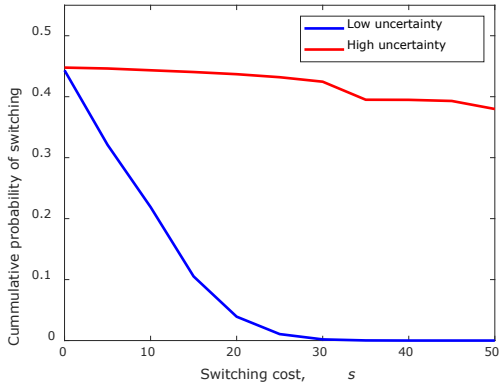


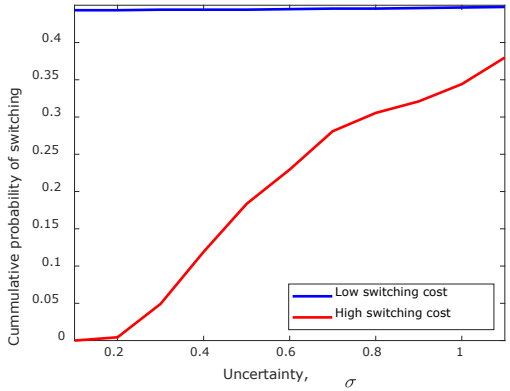
A. Use of the switching option when both options are present



B. Use of the growth option when both options are present

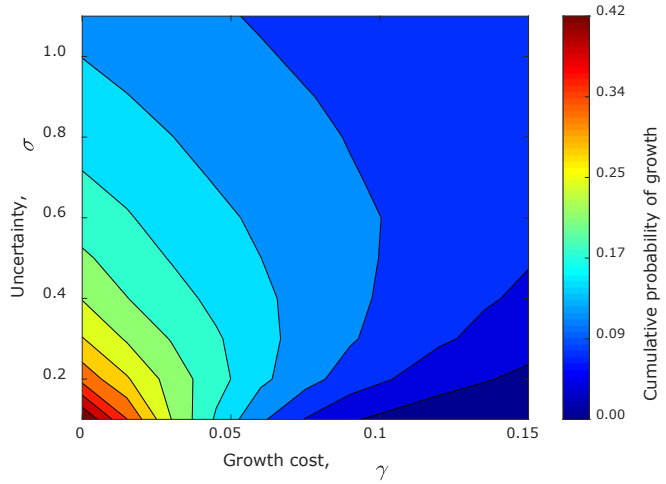




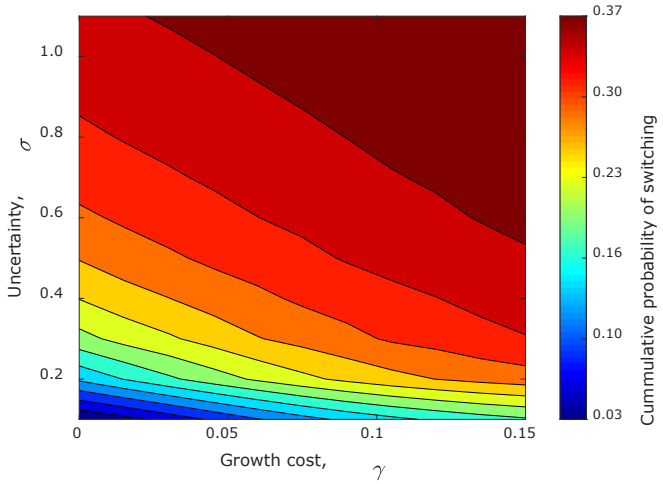


C. Use of the switching option when the growth option is absent

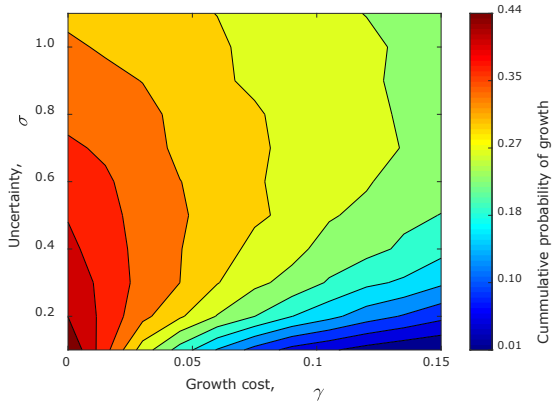
FIGURE 3. Implications of the switching cost and of uncertainty for the use of the two expansion options

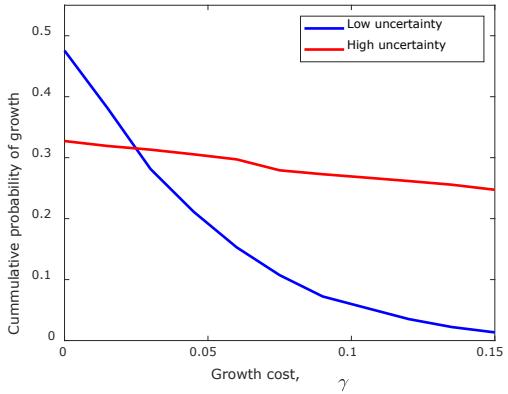


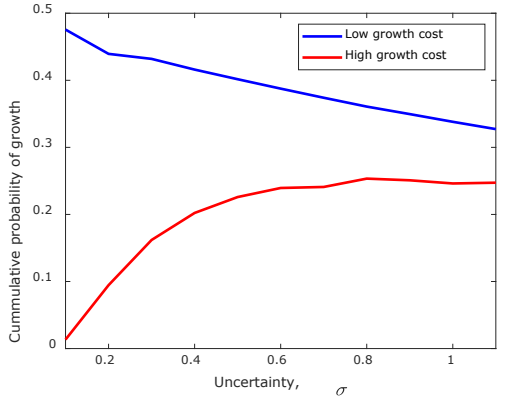
A. Use of the growth option when both options are present



B. Use of the switching option when both options are present

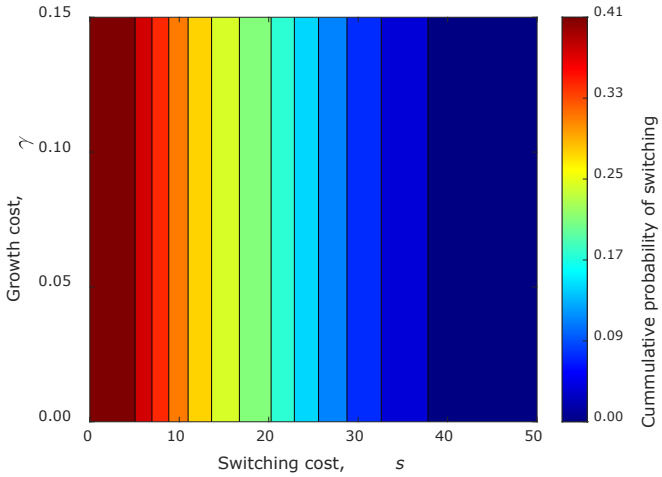




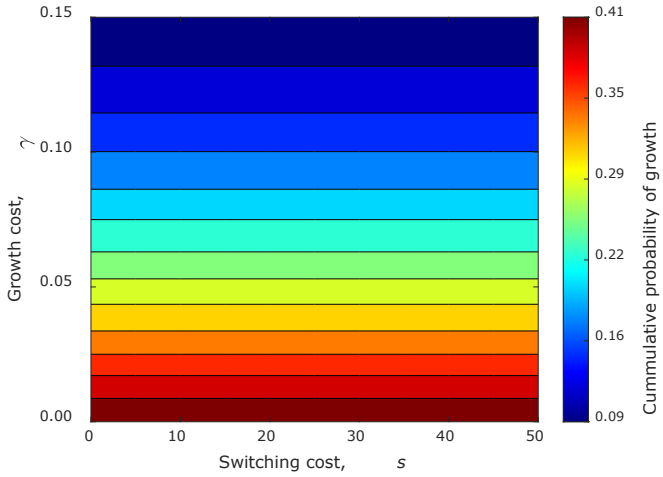


C. Use of the growth option when the switching option is absent

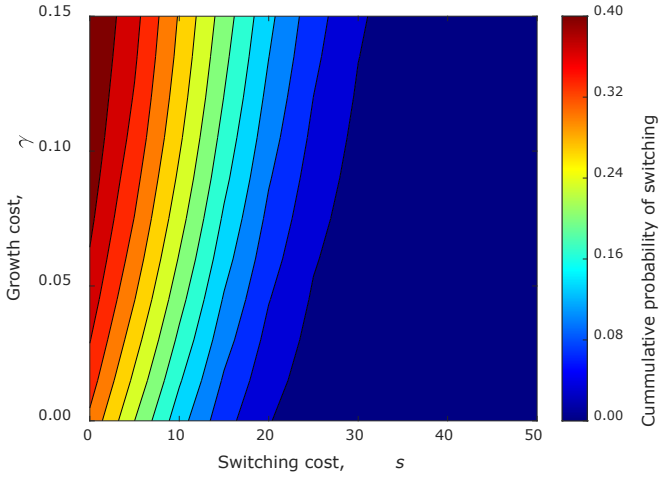
FIGURE 4. Implications of the growth cost and of uncertainty for the use of the two expansion options



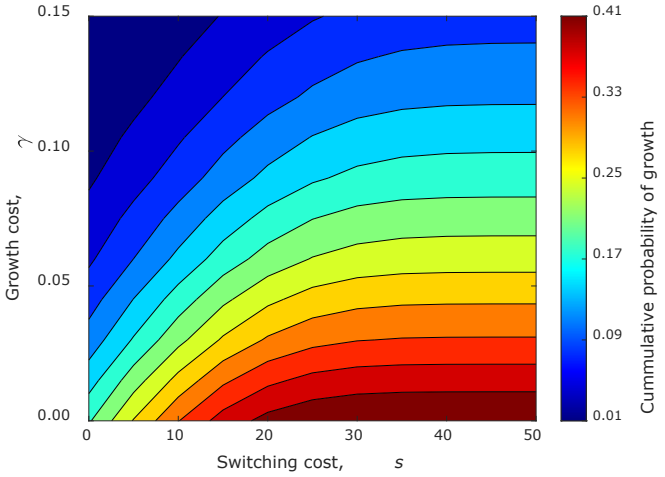
A. Use of the switching option when the growth option is absent



B. Use of the growth option when the switching option is absent



C. Use of the switching option when both options are present



D. Use of the growth option when both options are present

FIGURE 5. Implications of the switching cost and of the growth cost for the use of the two expansion options

Expansion Modes of Foreign Subsidiaries by Multinational Corporations

ONLINE APPENDIX: Estimation of option exercise choices

The switching and the growth options are not obligations for the MNC. They are exercised only if doing so makes the firm better off. A natural alternative available to the firm at any time is to continue holding the amount m_{i0} of resources in the foreign business i and the amount $(1-m_{i0})$ of resources in the domestic business j. When current realizations for C_{it} and C_{jt} are C_{it}^{x} and C_{jt}^{y} respectively, the expected net present value V_t^{xy0} for the firm that keeps its initial resource allocation can be expressed as follows:

$$V_t^{xy0} = m_{i0}C_{it}^{x} + (1-m_{i0})C_{it}^{y} + e^{-r\partial t}E^{P^{ij}}\left[V_{t+\partial t}^{xy} \mid \left(M_t^{*}=1,x,y\right)\right]. \quad \text{(A.1)}$$

In Equation A.1, $V_{t+\partial t}^{xy0}$ is the net present value of the firm in the immediate next time $(t+\partial t)$. Expectation $E^{P^{ij}}\left[V_{t+\partial t}^{xy} \mid \left(M_t^{*}=1,x,y\right)\right]$ is taken with respect to the probability distribution P^{ij} for C_{it} and C_{jt} and is conditioned on the current choice to keep the original resource allocation (i.e., $M_t^{*}=1$). The expectation is estimated when C_{it} and C_{jt} are in their respective states x and y.

Based on Equations 5 and 7 from the main text and based on Equation A.1 above, the firm's net present value V_t^{xy} can be stated as follows:

$$V_t^{xy} = \begin{bmatrix} \max\limits_{M_t} \begin{Bmatrix} m_{i0}C_{it}^{x} + (1-m_{i0})C_{it}^{y} + e^{-r\partial t}E^{P^{ij}}\left[V_{t+\partial t}^{xy} \mid \left(M_t^{*}=1,x,y\right)\right] & \text{if } M_{t-\partial t}=1, \\ -S_t^{x} + C_{it}^{x} + e^{-r\partial t}E^{P^{i}}\left[V_{t+\partial t}^{xS} \mid \left(M_t^{*}=2,x\right)\right] & \text{if } M_{t-\partial t}=1, \\ -G_t + 2m_{i0}C_{it}^{x} + (1-m_{i0})C_{jt}^{y} + e^{-r\partial t}E^{P^{ij}}\left[V_{t+\partial t}^{xyG} \mid \left(M_t^{*}=3,x,y\right)\right] & \text{if } M_{t-\partial t}=1 \end{Bmatrix} \\ C_{it}^{x} + e^{-r\partial t}E^{P^{i}}\left[V_{t+\partial t}^{xS} \mid \left(M_t^{*}=2,x\right)\right] \quad \text{if } M_{t-\partial t}=2 \\ 2m_{i0}C_{it}^{x} + (1-m_{i0})C_{jt}^{y} + e^{-r\partial t}E^{P^{ij}}\left[V_{t+\partial t}^{xyG} \mid \left(M_t^{*}=3,x,y\right)\right] \quad \text{if } M_{t-\partial t}=3 \end{bmatrix}. \quad \text{(A.2)}$$

Whereas, the firm's respective current choice $M_t^{*} \in \{1,2,3\}$ is expressed in the following way:

$$\left(M_t^* \middle| M_{t-\partial t}\right) = \begin{bmatrix} \arg\max\limits_{M_t} \begin{Bmatrix} m_{i0}C_{it}^x + (1-m_{i0})C_{it}^y + e^{-r\partial t}E^{P^{ij}}\left[V_{t+\partial t}^{xy} \middle| \left(M_t^* = 1, x, y\right)\right] & \text{if } M_{t-\partial t} = 1, \\ -S_t^x + C_{it}^x + e^{-r\partial t}E^{P^{ij}}\left[V_{t+\partial t}^{xS} \middle| \left(M_t^* = 2, x\right)\right] & \text{if } M_{t-\partial t} = 1, \\ -G_t + 2m_{i0}C_{it}^x + (1-m_{i0})C_{jt}^y + e^{-r\partial t}E^{P^{ij}}\left[V_{t+\partial t}^{xyG} \middle| \left(M_t^* = 3, x, y\right)\right] & \text{if } M_{t-\partial t} = 1 \end{Bmatrix} \\ 2 \quad \text{if } M_{t-\partial t} = 2 \\ 3 \quad \text{if } M_{t-\partial t} = 3 \end{bmatrix}. \quad \text{(A.3)}$$

Equations A.2 and A.3 are variants of the same Bellman equation (Bellman, 1957) that represents the firm's choice to expand its foreign business as dynamically optimal. Dynamic optimality demands that the firm choose the best time to exercise an option. This setting makes the firm compare: (a) the value of continuing to hold both expansion options if none of them has been exercised yet (i.e., the first lines in Equations A.2 and A.3), (b) the value of exercising the switching option if none of the two expansion options has been exercised yet (i.e., the second lines in Equations A.2 and A.3), and (c) the value of exercising the growth option if none of the two expansion options has been exercised yet (i.e., the third lines in Equations A.2 and A.3). If the firm already exercised the switching option or the growth option in the past, the firm stays in the corresponding new mode as reflected in the last two lines in Equations A.2 and A.3. To capture the possibility that one of the options was exercised in the past, Equation A.3 states the optimal decision $\left(M_t^* \middle| M_{t-\partial t}\right)$ at time t as conditional on what the firm did in the past.

Equations A.2 and A.3 split the problem of the expansion of the firm's foreign business into a sequence of sub-problems that are amenable to a numerical solution. The choice to expand the foreign business is expressed in a recursive form that relies on backward induction to derive optimal conditional choices $\left(M_t^* \middle| M_{t-\partial t}\right)$ at all times t and with all values of C_{it}^x, and C_{jt}^y. The solution involves the discretization of the continuous-time distribution P^{ij} specified with Equations 1–3. Like Sakhartov & Folta (2015), this model follows Boyle, Evnine & Gibbs

(1989) to discretize Equations 1–3 from the main text with a binomial lattice, which preserves the mean and the variance of the distribution if the number of time discretization steps N is sufficiently large and, thus, each time step $\partial t = T / N$ on the lattice is sufficiently short. On this lattice, the next-period margins $C_{it+\partial t}$ and $C_{jt+\partial t}$ have four states: $C^u_{it+\partial t}$ and $C^u_{jt+\partial t}$ with probability q^{uu}; $C^u_{it+\partial t}$ and $C^d_{jt+\partial t}$ with probability q^{ud}; $C^d_{it+\partial t}$ and $C^u_{jt+\partial t}$ with probability q^{du}; or $C^d_{it+\partial t}$ and $C^d_{jt+\partial t}$ with probability q^{dd}. The formulas for the involved discretization parameters that are based on Boyle et al. (1989) are provided below:

$$C^u_{it+\partial t} = u_i C_{it} \qquad \text{(A.4)}$$

$$C^d_{it+\partial t} = d_i C_{it} \qquad \text{(A.5)}$$

$$u_i = e^{\sigma_i \partial t} \qquad \text{(A.6)}$$

$$d_i = 1/u_i \qquad \text{(A.7)}$$

$$C^u_{jt+\partial t} = u_j C_{jt} \qquad \text{(A.8)}$$

$$C^d_{jt+\partial t} = d_j C_{jt} \qquad \text{(A.9)}$$

$$u_j = e^{\sigma_j \partial t} \qquad \text{(A.10)}$$

$$d_j = 1/u_j \qquad \text{(A.11)}$$

$$q^{uu} = 0.25\left\{1 + \rho + \sqrt{\partial t}\left[\left(r - 0.5\sigma_i^2\right)/\sigma_i + \left(r - 0.5\sigma_j^2\right)/\sigma_j\right]\right\} \quad \text{(A.12)}$$

$$q^{ud} = 0.25\left\{1 - \rho + \sqrt{\partial t}\left[\left(r - 0.5\sigma_i^2\right)/\sigma_i - \left(r - 0.5\sigma_j^2\right)/\sigma_j\right]\right\} \qquad \text{(A.13)}$$

$$q^{du} = 0.25\left\{1 - \rho + \sqrt{\partial t}\left[-\left(r - 0.5\sigma_i^2\right)/\sigma_i + \left(r - 0.5\sigma_j^2\right)/\sigma_j\right]\right\} \qquad \text{(A.14)}$$

$$q^{dd} = 0.25\left\{1 + \rho + \sqrt{\partial t}\left[-\left(r - 0.5\sigma_i^2\right)/\sigma_i - \left(r - 0.5\sigma_j^2\right)/\sigma_j\right]\right\}. \qquad \text{(A.15)}$$

Accordingly, expected values in the first three lines in Equations A.2 and A.3 can be estimated as follows:

$$E\left[V_{t+\partial t}^{xy0}\right] = q^{uu}V_{t+\partial t}^{uu0} + q^{ud}V_{t+\partial t}^{ud0} + q^{du}V_{t+\partial t}^{du0} + q^{dd}V_{t+\partial t}^{dd0} \qquad \text{(A.16)}$$

$$E\left[V_{t+\partial t}^{xS}\right] = (q^{uu} + q^{ud})V_{t+\partial t}^{uS} + (q^{du} + q^{dd})V_{t+\partial t}^{dS} \qquad \text{(A.17)}$$

$$E\left[V_{t+\partial t}^{xyG}\right] = q^{uu}V_{t+\partial t}^{uuG} + q^{ud}V_{t+\partial t}^{udG} + q^{du}V_{t+\partial t}^{duG} + q^{dd}V_{t+\partial t}^{ddG}, \qquad \text{(A.18)}$$

where $V_{t+\partial t}^{uu0}$ and $V_{t+\partial t}^{uuG}$ are calculated using $C_{it+\partial t}^{u}$ and $C_{jt+\partial t}^{u}$; $V_{t+\partial t}^{ud0}$ and $V_{t+\partial t}^{udG}$ are estimated using $C_{it+\partial t}^{u}$ and $C_{jt+\partial t}^{d}$; $V_{t+\partial t}^{du0}$ and $V_{t+\partial t}^{duG}$ are assessed using $C_{it+\partial t}^{d}$ and $C_{jt+\partial t}^{u}$; and $V_{t+\partial t}^{dd0}$ and $V_{t+\partial t}^{ddG}$ are computed using $C_{it+\partial t}^{d}$ and $C_{jt+\partial t}^{d}$. In turn, $V_{t+\partial t}^{uS}$ is estimated using $C_{it+\partial t}^{u}$, whereas $V_{t+\partial t}^{dS}$ is evaluated using $C_{it+\partial t}^{d}$.

Like in existing option valuation models (e.g., Sakhartov &Folta, 2015), the backward induction procedure starts at the penultimate time $t = T - \partial t$ with the terminal conditions $V_T^{xy0} = 0$, $V_T^{xS} = 0$, and $V_T^{xyG} = 0$ suggesting that the resources will have fully exhausted their ability to generate returns by that time. The algorithm proceeds recursively backward in time with a step ∂t until it reaches time $t = 0$. At this point in the estimation, the model returns the net present value of the firm V_0^{xy}, but the firm's choices are still characterized as conditional $\left(M_t^* \middle| M_{t-\partial t}\right)$. The next stage of the model represents a step that goes beyond the backward induction and is more analytically complicated. In particular, because the firm is known to have initially split its resources equally between its domestic and foreign businesses ($M_0 = 1$), the model can now change the direction for going through the lattice and follows recursively forward in time until it reaches time $t = T$. In each step going forward in time and for each combination

of margins C_{it}^{x}, and C_{jt}^{y}, the model derives unconditional choice M_t^* based on the known immediate previous choice $M_{t-\partial t}$ and on the optimal conditional decision $\left(M_t^* \middle| M_{t-\partial t}\right)$ recovered with the backward induction. Finally, the three-dimensional matrix (t, x, and y) that is generated for M_t^* enables the analyses reported in the main text.

Summary of the step-by-step procedure for estimating corporate risk

1. Select a sufficiently large number of the time discretization steps N. Like Sakhartov & Folta (2015), this study uses $N = 200$. Because the binomial approximation of the geometric Brownian motion is known to make the option value estimated with that approximation converge from below to the true option value with the increase of the time discretization steps, the common heuristic is to increase N until the respective changes in the estimated option value get insignificant. An additional constraint on N is that the used value of N should keep the transition probabilities estimated with Equations A.12-A.15 positive.
2. Using Equations A.4-A.11, create the bivariate binomial lattice. This lattice should be filled in with all values of C_{it} and C_{jt} over time $t \in [0, T)$.
3. Using Equations A.12-A.15, estimate the transitional probabilities q^{uu}, q^{ud}, q^{du}, and q^{dd}.
4. Set terminal values $V_T^{xy0} = 0$, $V_T^{xS} = 0$, and $V_T^{xyG} = 0$.
5. Start at the penultimate time $t = T - \partial t$ and, using Equation A.2, estimate $V_{T-\partial t}^{xy}$ for all feasible combinations of $M_{T-\partial t}$ (i.e., the current choice) and $M_{T-2\partial t}$ (i.e., the immediate

previous choice) and on all nodes of the lattice feasible at time $t = T - \partial t$ (i.e., $C^{x}_{iT-\partial t}$ and $C^{y}_{jT-\partial t}$).

6. For each feasible value of $M_{T-2\partial t}$ (i.e., the immediate previous choice) considered in Step 5, solve Equations A.2 and A.3 with respect to $M_{T-\partial t}$ (i.e., the current choice) considered in Step 5. Store all the conditional solutions $\left(M^{*}_{T-\partial t} \middle| M_{T-2\partial t}\right)$ and $V^{xy}_{T-\partial t}\left(M^{*}_{T-\partial t} \middle| M_{T-2\partial t}\right)$.
7. Proceed to the immediate previous time $t = T - 2\partial t$. Use $V^{xy}_{T-\partial t}\left(M^{*}_{T-\partial t} \middle| M_{T-2\partial t}\right)$'s estimated in Step 6 and, using Equation A.2, estimate $V^{xy}_{T-2\partial t}\left(M_{T-2\partial t} \mid M_{T-3\partial t}\right)$ for all feasible combinations of $M_{T-2\partial t}$ (i.e., the new current choice) and $M_{T-3\partial t}$ (i.e., the new immediate previous choice) and on all nodes of the lattice feasible at time $t = T - 2\partial t$ (i.e., $C^{x}_{iT-2\partial t}$ and $C^{y}_{jT-2\partial t}$).
8. For each feasible value of $M_{T-3\partial t}$ (i.e., the immediate previous choice) considered in Step 7, solve Equations A.2 and A.3 with respect to $M_{T-2\partial t}$ (i.e., the current choice) considered in Step 7. Store all the conditional solutions $\left(M^{*}_{T-2\partial t} \middle| M_{T-3\partial t}\right)$ and $V^{xy}_{T-2\partial t}\left(M^{*}_{T-2\partial t} \middle| M_{T-3\partial t}\right)$.
9. Procced with Steps 7 and 8 recursively backward in time down to time $t = 0$. When time $t = 0$ is reached as the current time, the backward induction is completed. The stored values of $\left(M^{*}_{t} \middle| M_{t-\partial t}\right)$ for $t \in [0, T - \partial t]$ represent the main outcome of Steps 4-9 (i.e., backward induction) to be used in next steps.

The next stage of the model represents steps that go beyond the backward induction and are more analytically complicated and much more computationally intense. In particular, because the firm

is known to have initially split its resources equally between its domestic and foreign businesses ($M_{0-\partial t}=1$), the model can now change the direction for going through the lattice and follows recursively forward in time until it reaches time $t=T$.

10. Using the known value $M_{0-\partial t}=1$ and the stored values $\left(M_0^*\middle|M_{0-\partial t}\right)$, identify the unconditional choice M_0^* that maximizes V_0^{xy} in Equation A.2. Because $t=0$ there is no uncertainty in Equations 1-3, the probability of that unconditional choice equals one.
11. Go to the immediate next time $t=0+\partial t$, in which there may be four scenarios (i.e., nodes on the lattice) for the two margins: $C_{i0+\partial t}^u$ and $C_{j0+\partial t}^u$ with probability q^{uu}; $C_{i0+\partial t}^u$ and $C_{j0+\partial t}^d$ with probability q^{ud}; $C_{i0+\partial t}^d$ and $C_{j0+\partial t}^u$ with probability q^{du}; and $C_{i0+\partial t}^d$ and $C_{j0+\partial t}^d$ with probability q^{dd}. Use M_0^* derived in Step 10 and, in each of these four scenarios, derive $M_{0+\partial t}^*$ that maximizes $V_{0+\partial t}^{xy}$ in Equation A.2. Based on the known probabilities q^{uu}, q^{ud}, q^{du}, and q^{dd} and based on the derived choices $M_{0+\partial t}^*$ on each respective node of the lattice, store probabilities $p^{xy}\left(M_{0+\partial t}^*=1\right)$, $p^{xy}\left(M_{0+\partial t}^*=2\right)$, and $p^{xy}\left(M_{0+\partial t}^*=3\right)$. For example, if on node "*ud*" $M_{0+\partial t}^*=2$: $p^{ud}\left(M_{0+\partial t}^*=2\right)=q^{ud}$, $p^{ud}\left(M_{0+\partial t}^*=1\right)=0$, and $p^{ud}\left(M_{0+\partial t}^*=1\right)=0$.
12. Go to the immediate next time $t=0+2\partial t$ that has nine scenarios which follow the four scenarios at time $t=0+\partial t$. Some scenarios at time $t=0+2\partial t$ can be entered from more than one scenario at time $t=0+\partial t$. Furthermore, because different scenarios at time $t=0+\partial t$ may have different values of $M_{0+\partial t}$, different paths from $t=0+\partial t$ into $t=0+2\partial t$ may have different optimal solutions $\left(M_{0+2\partial t}^*\middle|M_{0+\partial t}\right)$. Therefore, all possible

paths into the node, for which probabilities $p^{xy}\left(M^{*}_{0+2\partial t}=1\right)$, $p^{xy}\left(M^{*}_{0+2\partial t}=2\right)$, and $p^{xy}\left(M^{*}_{0+2\partial t}=3\right)$ are assessed, need to be accounted for and scaled by their probabilities. The resulting products should be summed on each node "*xy*" and separately for each possible outcome $M^{*}_{0+2\partial t}=1$, $M^{*}_{0+2\partial t}=2$, and $M^{*}_{0+2\partial t}=3$.

13. Proceed further recursively forward in time from time $t=0+3\partial t$ to time $t=T-\partial t$ and repeat Step 12 to receive $p^{xy}\left(M^{*}_{t}=1\right)$, $p^{xy}\left(M^{*}_{t}=2\right)$, and $p^{xy}\left(M^{*}_{t}=3\right)$ on all nodes "*xy*" of the lattice and in all times $t\in\left[0+3\partial t, T-\partial t\right]$.
14. Step 13 leads to the creation of three matrices: (i) for $p^{xy}\left(M^{*}_{t}=1\right)$, (ii) for $p^{xy}\left(M^{*}_{t}=2\right)$, and (iii) for $p^{xy}\left(M^{*}_{t}=3\right)$. Each of these matrices is three-dimensional, with the dimensions being *x*, *y*, and *t*.
15. Figures 1 and 2 use the matrices for $p^{xy}\left(M^{*}_{t}=2\right)$ and for $p^{xy}\left(M^{*}_{t}=3\right)$ as they are formed in Step 14 because each panel in these figures actually shows the probability of using an expansion option against all possible values of *x* and *y*, in a particular time *t*.
16. In all other figures, the data from the matrices are processed in the following way. The cumulative probability of using the switching option by the middle of the lifecycle of the resources is $\sum_{t=0}^{t=100\partial t}\sum_{x}\sum_{y}p^{xy}\left(M^{*}_{t}=2\right)$, whereas the cumulative probability of using the growth option by the middle of the lifecycle of the resources is $\sum_{t=0}^{t=100\partial t}\sum_{x}\sum_{y}p^{xy}\left(M^{*}_{t}=3\right)$.