Pricing in Service Platforms: Who Should Set the Prices?

Gérard P. Cachon, Tolga Dizdarer, Gerry Tsoukalas*

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Abstract

Service platforms like Uber and AirBnb create markets that connect customers with service providers. Platforms provide the medium of interaction and receive commissions from each transaction taking place between the two parties. The servers working for the platforms are commonly flexible in choosing how much they work. While some platforms like AirBnb further provide the servers the flexibility of choosing their own fares, others like Uber and Lyft assume the role of market-maker and set the prices for the whole market. What should be the extent of platform's role in setting the prices in the market is a fundamental question for the platform and we study it as a trade-off between price efficiency and information utilization. In a market with cost information asymmetry, server pricing allows platform to utilize servers' private cost information and increase server participation. However, this policy is risky. As the prices are determined by self-interested actors, the market is susceptible to two well-known pricing inefficiencies observed in decentralized markets: inflation of prices due to double-marginalization and deflation of prices due to competition. If the market competition is at an extreme, server pricing may significantly shrink the market size and damage platform's profits. Platform pricing is a robust policy that alleviates the prices efficiencies; however, it cannot capitalize on servers' cost information and results in non-optimal profits for the platform. We find that offering quantity bonuses/surcharges under server pricing eliminates price inefficiencies and achieves optimal profits for the platform.

1 Introduction

Service platforms design marketplaces that act as a medium between independent service providers and potential customers. This concept has proven to be relevant for a diverse range of markets, and

*OID Department, The Wharton School, University of Pennsylvania, Philadelphia, PA 19104. Cachon: cachon@wharton.upenn.edu, Dizdarer: dizdarer@wharton.upenn.edu, Tsoukalas: gtsouk@wharton.upenn.edu.

various platforms have found their way into the economy. Platforms like Airbnb and CouchSurfing allow home-owners to rent their homes to interested parties, eBay and Etsy allow e-commerce sellers to connect with customers, and Uber and BlaBlaCar allow car-owners to share their car rides with passengers. The value these platforms bring to the market is the efficient matching of supply and demand. A platform's efficiency relies on its ability to coordinate both sides of the market. When it acts as a market-maker, the platform may achieve this by regulating the prices: increasing prices under high-demand periods and decreasing prices under low-demand periods. However, the robustness of this approach is limited by the extent of platform's information. When servers in a market have private information, the platform's choice of prices may not be optimal. Allowing servers to set their own prices can utilize servers' information; however, this system's efficiency depends on the platform's ability to coordinate self-interested actors without direct price control.

Motivated by this observation, we study a primary question for the platform: Is it preferable for a platform to control the prices or is the platform better off allowing the server to set their own prices? Since the platforms are generally exposed to information asymmetries and composed of self-interested actors, it is important to understand under which cases the platform can let go of the pricing control and still function effectively. A relevant follow-up question we answer is: How can a platform ensure that the market is not exposed to coordination issues when servers set their own prices?

These questions are applicable not only for new startups that want to follow the best practices, but also for established platforms that need to adapt to changing dynamics of service economies. A recent development that has affected the service platforms is the adoption of AB5 in California. This state statute emphasizes a contractor's freedom to do its business without the company's control and direction. Freedom in setting prices has notably been considered a pre-requisite of this description (Bhuiyan 2020), and there have been researches that argue that granting price freedom to drivers is a necessary measure for platforms like Uber and Lyft to continue classifying their drivers as contractors (Paul 2016). Discussions around Uber's pricing power are not limited to California. In the UK, a recent court ruling granted Uber drivers employee status. This decision is based on Uber's control over fares and the contractual terms it enforces to the drivers (O'Brien 2021). In parallel to these developments, Uber has started experimenting with a feature that lets drivers set their own fares (Rana 2020). This feature was later extended to all drivers in California (Uber 2020). This highlights the relevance of server pricing for established platforms like Uber who may be considering changes to their pricing strategies to confront new employment classification laws. Our research answers another question that might have implications for policy-makers in this context: Is a platform's control over prices detrimental to servers' profits?

Another development that motivates our research is the emergence of new platforms that use blockchain technology to decentralize their business models. In ride-sharing market, Drife and Arcade City, in rental markets, Bee Token, are some platforms that have recently emerged in this respect. These platforms utilize blockchain technology and smart contracts to build systems that function efficiently without a central coordinator. By decentralizing tasks such as information storage and dissemination, and other operational decisions, these platforms grant greater autonomy to the agents using the platform. Some of these platforms further utilize tokens, which are digital assets that are used for monetary or non-monetary exchanges within the platform's ecosystem. Although the use of tokens is beyond our research scope, these devices provide novel ways for platforms to interact with other agents and coordinate their actions. As the research on decentralizing pricing decision in the context of service platforms is limited, we find it important to understand what mechanisms a platform has in its disposal to coordinate a decentralized organization.

From an operational standpoint, the argument for servers to set their own prices stems from supply-side heterogeneity. The servers in a market may provide differentiated services and encounter different marginal costs of serving. This is especially true for service platforms due to their flexible work arrangements, which attract contractors with high time-variation in their opportunity costs. For example, see Chen et al. (2019) for evidence of reservation wage variations among Uber servers. In these markets, platform pricing may not sufficiently utilize servers' private information and decrease both the platform and server profits. Despite this limitation, platforms may prefer to set the prices due to its ability to regulate the market. When a central coordinator sets the prices, the platform can ensure that prices are not subject to coordination issues.

In this paper, we model a service platform mediating transactions between servers and customers. We study two pricing policies platform can adopt: platform pricing, where the platform sets the prices for all servers, and server pricing, where prices are defined by the competitive equilibrium of server decisions. We identify the coordination inefficiencies that can arise in a decentralized arrangement and study the trade-off between the two pricing policies. Our main results are as follows. In a market where servers have private information about their serving costs, the decision between platform- and server pricing is a trade-off between price coordination and information utilization. Platform pricing is a robust policy that generates a balance between supply and demand. Server pricing can further promote server entry by allowing servers to tailor prices according to their marginal costs; however, it may not be desirable if servers are over- or under-concerned with their own competitiveness within the market. Under very strong competition, server pricing may lead to "race to the bottom", which harms server entry and decreases platform profits. When competition is very weak, large server entry under server pricing can be accompanied with excessively high prices, in which case both platform and server profits suffer. Under both extremes of market competition, platform prefers platform pricing due to its ability to set "coordinated" prices. Regardless of the pricing policy used, the servers' total profits is a fixed fraction of the platform's profits. Therefore, there is no incentive misalignment between the platform and the servers when choosing the pricing policy. The policy that maximizes platform's profits also maximizes server profits.

We also look at platform's optimal mechanism design. The platform's optimal contract achieves strictly higher profits than platform pricing and server pricing almost everywhere. This contract can be replicated through a quantity bonus scheme where servers get fee discounts for serving larger demands. By allowing servers to set their own prices, this contract can attract a large supply without experiencing price inefficiencies. Under this mechanism, platform achieves price coordination by manipulating servers' competitive incentives.

2 Literature Review

Our research is connected to three literature streams: vertical integration/contracts in supply chains, operational controls in service platforms, and design of decentralized markets.

The first stream studies price coordination in the context of vertical integration, e.g., the seminal work of Spengler (1950) shows that vertical integration can effectively coordinate the market by eliminating double marginalization. In subsequent work, Dixit (1983), Rey and Tirole (1986), Deneckere et al. (1996) show how price controls, in the form of *resale price maintenance*, can achieve profits that replicate vertical integration. This stream identifies price inefficiencies associated with decentralized channels and provides motivation for our research. Our paper diverges from this stream by examining indirect forms of control, e.g. through the terms of the platform's commission structure.

There is a large literature on contracting to coordinate supply chains (e.g. Pasternack 1985, Padmanabhan and Png 1997, Dana and Spier 2001, Cachon and Lariviere 2005, Song et al. 2008). Most relevant to our setting are the papers exploring cost information asymmetry. In this respect, Corbett and De Groote (2000), Ha (2001), Corbett et al. (2004), Mukhopadhyay et al. (2008), Yao et al. (2008), Xie et al. (2014), Ma et al. (2017) look at contracts a monopolistic supplier offers to buyer(s) with private cost information. Our paper extends this line of research by studying price coordination in the context of a service platform. By moving away from a supply chain setting with manufacturer and retailers, our work explores a market without a vertical relationship between actors. In our model, the services do not flow in a linear fashion and there is no intermediary good exchange between platforms and other agents.

The research on price coordination in vertical disintegrated platforms is sparse. Close to our setting, Aymanns et al. (2020) study a vertically disintegrated payment platform where price is set through the competitive equilibrium of service processors. The authors find that vertical disintegration generally lowers consumer welfare. Unlike us, the authors do not study how market competition affects price efficiency, which is a pivotal theme in our work.

There are also papers that study the use of quantity bonuses/discounts as incentive mechanisms. Lal and Staelin (1984), Monahan (1984), Dolan (1987), Dada and Srikanth (1987), Weng (1995), Corbett and De Groote (2000) are some of the early examples. Unlike us, these papers focus on the use of quantity bonuses in vertical supply chains. In the context of service platforms, Liu et al. (2019) study a market with two competing platforms and find that paying bonuses along with the commission contract can increase service provider retention. Our paper provides another motivation for platforms to offer quantity bonuses by illustrating its use as a price coordination mechanism.

Our research is also related to the line of literature that deals with operational decisions within service platforms. This literature deals with a platform's decisions over prices, fees or other mechanisms to understand how the platform can influence flexible workers without imposing direct control. In this domain, Gurvich et al. (2016), Cachon et al. (2017), Bai et al. (2018), Taylor (2018), Hu and Zhou (2019) study optimal pricing and fee structures in the presence of flexible workers when demand is uncertain and servers are heterogeneous in their opportunity costs. Although these papers study pricing decisions within platforms, unlike our work, they do not explore decentralized pricing.

A few other papers deal with other types of operational control in service platforms when the pricing decision is not centralized. In this context, Allon et al. (2012), Kanoria and Saban (2017), Bimpikis et al. (2020), Arnosti et al. (2021) look at information design in a market where prices are endogenously controlled by one side of the market. Unlike these papers, we focus specifically on platform's control over its fee structure to understand its use for price coordination. A paper that, like us, studies platform's commission structure is Birge et al. (2020). The authors assume prices are endogenously determined by a competitive equilibrium of sellers that are heterogeneous in their types. Focusing on the network structure and compatibility of buyers and sellers, they find that the platform should charge different commissions to different types of sellers to earn optimal profits. Unlike our paper, their primary focus is on network structures, so they do not explore the trade-offs associated with centralizing vs. decentralizing pricing decisions.

Our paper also contributes to growing body of research on the design of decentralized market/economies, some of which belong outside the Operations field. In Economics literature Malamud and Rostek (2017), Üslü (2019), Bodoh-Creed et al. (2021), in Finance literature Hagströmer and Menkveld (2019), in Marketing literature Li et al. (2019), in Management literature Chen et al. (2020) are some of the recent work on decentralized markets. In the Operations field, Hagiu and Wright (2015), Tsoukalas and Falk (2020), Hau et al. (2021), Ke and Zhu (2021) look at various operational controls within decentralized platforms, although they do not study price coordination.

To our knowledge, our paper is the first to study how market competition affects a service platform's decision to centralize or decentralize the pricing control. We contribute to the literature on decentralized markets by identifying pricing inefficiencies that can occur in decentralized platforms and finding contractual mechanisms that can ensure coordination in the system. We also contribute to the practice by identifying the advantages/limitations of these pricing policies and describing their market implications.

3 Model

We model a platform that mediates transactions between customers and servers. The platform has only indirect control over both actors, e.g., by setting policies that affect their behavior. There is a unit mass of servers defined on a continuum $(0, 1)$. We model server decisions as a mean field game. Each server in the server pool is infinitesimal and is affected only by averaged metrics characterized by the equilibrium decisions of all servers.

Servers have heterogeneous costs described by distribution $F(c)$ ~Uniform$(0, 1)$. These costs capture various factors such as opportunity costs, time-preferences or other expenses, which vary within the market. Each server's cost realization is private information. The other servers and the platform only observe the distribution.

Let $p(c)$ be the price of a server with serving cost $c \in (0, 1)$. We refer to this server as server c. The servers' prices affect the customer demand they serve both directly and indirectly through the (demand-weighted) average market price, $\overline{p}$. In particular, a server's demand is defined as:

$$q(c) = 1 - \beta\overline{p} + \gamma(\overline{p} - p(c)), \tag{1}$$

with

$$\overline{p} = \frac{\int_0^{\hat{c}} q(c)p(c)\ dc}{\int_0^{\hat{c}} q(c)\ dc}. \tag{2}$$

The demand function consists of two components: The first, $1-\beta\overline{p}$, is the market size component and captures the increase (decrease) in market size as the average market price falls (increases). The parameter β determines customer's sensitivity to average market price and we term it as "platform attractiveness parameter". The second, $\gamma(\overline{p} - p(c))$, is the market competition component and captures the increase (decrease) in a server's demand when setting a lower (higher) price relative to the market average. The parameter γ regulates the strength of competition among the servers. We term this parameter as "server competition parameter".

Both components of demand are influenced by the average market price. Whether a server's demand increases or decreases in the average price depends on the relative size of γ and β. If $\gamma > \beta$, the competition component is relatively large. In this case, servers prefer a high average price and we consider this as a competitive market. If $\gamma = \beta$, average market price doesn't influence server demands. In this case, all servers behave as monopolies. If $\gamma < \beta$, server demands decrease in the average market price. Due servers' preference for a small average price, we consider this as a setting with weak market competition. At the extreme, $\gamma = 0$, the servers' demands are perfectly inelastic in their own prices. In this case, the competition component of demand completely vanishes, each server's demand depends only on the market average. Conversely, if $\gamma \to \infty$, the demand function collapses to Bertrand competition, where only the server(s) with the lowest price serve the customers.

Inspired, among other things, by Uber's pricing experiments in California (as described in the introduction), the platform may either impose a price on the servers—"centralized platform pricing"—or allow them to set their own prices—"decentralized server pricing". As a benchmark, we will also look at pricing from a central coordinator's perspective—"coordinated pricing". Regardless of the setting, we will start by assuming the platform utilizes a commission contract, that is, the servers pay ϕ portion of their earnings to the platform as a fee (for example, in the case of Uber, this fee is around 20%). We will explore alternative contracts subsequently. In all cases, a server with a price $p(c)$ earns a profit of

$$\pi(c) = q(c)((1-\phi)p(c) - c). \tag{3}$$

We assume servers are profit-maximizing with zero outside option, so they participate only if their expected profit is non-negative. Let $\hat{c}$ be the highest cost server that participates. The platform's profits are given by

$$\Pi = \phi \int_{0}^{\hat{c}} q(c)p(c)\ dc = \phi\overline{p} \int_{0}^{\hat{c}} q(c)\ dc. \tag{4}$$

Next, we describe how these expressions change for each pricing policy.

Pricing policies

In all cases, there are two decision periods and the sequence of events is presented in Figures 1 and 2 respectively.

Centralized platform pricing, Figure 1, is similar to the pricing policies often used by firms like Uber and Lyft. Both the platform and servers know the distribution of costs, but only servers can observe their own cost realization. For now, we assume that in the first decision period, the platform sets a price, p, and commission, ϕ, which uniformly applies to all servers. In Section 5, we relax the uniformity assumption and examine optimal mechanism design using more tailored pricing policies. The servers observe these terms and their own cost realizations before making their participation decisions. In the second decision period, servers choose whether to participate or not participate. Subsequently, the service transactions between the servers and customers occur and the platform's profit is realized.

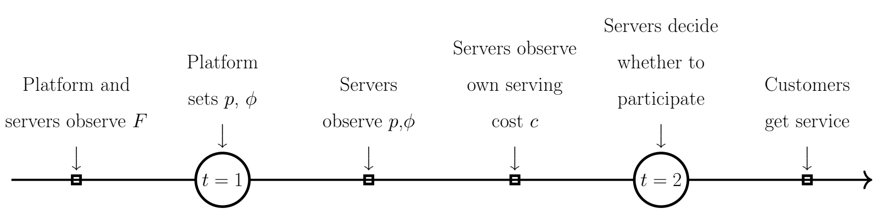


Figure 1: Platform pricing timeline

Decentralized server pricing, Figure 2, is often used by platforms like Airbnb and BlaBlaCar, though as previously discussed, ride-hailing platforms such as Uber have also experimented with this pricing policy in certain markets. Server pricing follows a similar sequence of events, but unlike platform pricing, the platform does not set the prices for the servers in the first period. Instead, servers make their participation and pricing decisions simultaneously in the second period. Each server sets the price that maximizes their own equilibrium profits and only servers with non-negative expected profits participate.

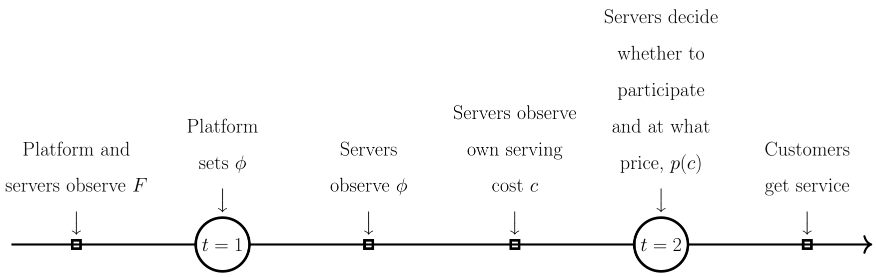


Figure 2: Server pricing timeline

Coordinated pricing, which we study as a benchmark policy, policy follows a similar timeline as server pricing. Unlike server pricing, we assume that a central coordinator sets the prices for all servers such that the total server profits is maximized.

4 Analysis of Pricing Policies

In this section, we analyze the optimal decisions of the agents under the different pricing policies and derive the corresponding market characteristics. First, we look at the equilibrium behavior under centralized platform pricing in Section 4.1. Then, we explore decentralized server pricing in Section 4.2. The benchmark case of pricing by a central coordinator is examined in Section 4.3. In Section 4.4, we discuss the trade-offs associated with the different pricing policies.

4.1 Centralized Platform Pricing

Under platform pricing, participating servers earn p and pay ϕp to the platform for every unit served. Conditional on entry, a server with cost c earns:

$$\begin{aligned} \pi(c) &= (1 - \beta p + \gamma(p - p))((1 - \phi)p - c) \\ &= (1 - \beta p)((1 - \phi)p - c). \end{aligned} \tag{5}$$

The server profits are decreasing in c. The highest cost participant is indifferent between participating and not participating:

$$\pi(\hat{c}) = 0 \quad \implies \quad \hat{c} = \hat{c}(p, \phi) = (1 - \phi)p. \tag{6}$$

The platform chooses the price and commission that maximizes its total profits assuming that server entry occurs as defined above. In this case, $\overline{p} = p$, implying $q(c) = 1 - \beta\overline{p}$, and the platform's optimization problem in (4) simplifies to:

$$\begin{aligned} \max_{\overline{p},\phi} \quad & \Pi^{\mathcal{P}} = \phi\overline{p} \int_{0}^{\hat{c}(\overline{p},\phi)} (1-\beta\overline{p})\, dc \\ \text{s.t.} \quad & \text{Eq. (6).} \end{aligned} \tag{7}$$

Proposition 1. *Under centralized platform pricing, there exists a unique solution to the platform's problem. The optimal platform pricing policy exhibits the following characteristics: (i) The platform's choice of optimal price and commission are* $p(c) = \frac{2}{3\beta}$, $\phi = \frac{1}{2}$. *(ii) The platform earns* $\Pi^{\mathcal{P}} = \frac{1}{27\beta^2}$. *(iii) The equilibrium mass of participating servers and the average price are* $\hat{c} = \frac{1}{3\beta}$, $\overline{p} = \frac{2}{3\beta}$. *(iv) Total quantity served in the market is* $Q = \frac{1}{9\beta}$.

The platform's optimal decisions and earnings do not depend on the competition parameter, γ. All servers are attained the same prices. They serve the same demand and pay half of their revenues to the platform as fees. When choosing the price, the platform considers a trade-off between supply, which is increasing in price, and demand, which is decreasing in price. As platform becomes less attractive (β increases), the platform lowers its prices to attract more customers, which hurts server entry. Due to platform's susceptibility to changes in market size, platform's earning monotonically decreases in β. The advantage of platform pricing is its ability to eliminate any adverse price effects stemming from market competition. We discuss these effects in detail in the next section. Due to its insensitivity to market competition, we consider platform pricing to be a robust policy that achieves a (rough) balance between supply and demand.

4.2 Decentralized Server Pricing

Under server pricing, the platform sets its commission percentage, ϕ. Then, the servers simultaneously choose whether to participate and at what price. As servers are small actors, they choose the prices that maximize their individual profits, taking the equilibrium average market price, $\overline{p}$, as given. Server c has the following pricing problem:

$$\max_{p(c)} \quad \pi(c) = \big(1-\beta\overline{p}+\gamma(\overline{p}-p(c))\big)((1-\phi)p(c)-c), \tag{8}$$

This has solution

$$p(c) = \frac{1}{2\gamma}\left(1+(\gamma-\beta)\overline{p}+\frac{c\gamma}{1-\phi}\right). \tag{9}$$

Servers participate only if their profit is non-negative. The highest cost participant is:

$$\hat{c} = \frac{(1-\phi)(1+(\gamma-\beta)\overline{p})}{\gamma}. \tag{10}$$

The average market price, $\overline{p}$, is characterized as an equilibrium of server decisions governed by Equation (2) and is unique. The platform's objective is to choose the optimal commission percentage under the assumption that the servers reach the equilibrium defined above. In this case, the platform's optimization problem in (4) becomes

$$\begin{aligned} \max_{\phi} \quad & \Pi^{\mathcal{S}} = \phi \overline{p} \int_0^{\hat{c}} (1 - \beta \overline{p} + \gamma(\overline{p} - p(c)))\, dc \\ \text{s.t.} \quad & \text{Eq. } (2), (9), (10). \end{aligned} \tag{11}$$

Proposition 2. *Under decentralized server pricing, there exists a unique solution to the platform's problem. The optimal server pricing policy exhibits the following characteristics: (i) Servers' equilibrium price and platform's optimal commission are* $p(c) = \frac{3}{2(2\beta+\gamma)} + c$, $\phi = \frac{1}{2}$. *(ii) The platform earns* $\Pi = \frac{9}{8}\left(\frac{\gamma}{(2\beta+\gamma)^3}\right)$. *(iii) The equilibrium mass of participating servers and the average price are* $\hat{c} = \frac{3}{2(2\beta+\gamma)}$, $\overline{p} = \frac{2}{2\beta+\gamma}$. *(iv) Total quantity served in the market is* $Q = \frac{9}{8}\left(\frac{\gamma}{(2\beta+\gamma)^2}\right)$.

Under server pricing, the platform's commission choice does not depend on market parameters, but its earnings do. As servers are concerned with both platform's attractiveness and their own competitiveness, their pricing decisions are sensitive to both market conditions. If market attractiveness is low (β is high), the servers set low prices to capture demand. The servers also adjust their prices according to market competitiveness. The earnings under server pricing are maximized if $\gamma = \beta$, at which servers behave as monopolies. As the competition parameter deviates away from β, platform earnings monotonically decrease. Under the two extremes (eg. $\gamma \to 0$, $\gamma \to \infty$) the platform's earnings approach zero. In these cases, server pricing exhibits coordination inefficiencies that lead to excessively low or high prices.

In particular, as the competition parameter increases, customers' sensitivity to a server's price increases. Self-interested servers respond to this by lowering their prices. If market competition is very strong, the average market price gets too low and leaves high-cost servers unable to profitably participate. The platform cannot mitigate this through a hike in commissions. Increasing commissions increases the prices of the participating servers, which should mitigate the competition effect; however, this behavior is also complemented with the exit of high-price servers. These two counteracting effects balance each other out and the average price remains unchanged. This *race to the bottom* may leave the platform exposed to coordination failures. Since the platform doesn't have any tools at its disposal to effectively stop this price decrease, this results in a significant reduction of profits. We refer to this phenomenon as the *destructive competition effect.*

Conversely, if the competition parameter is low, customers are insensitive to price variations in the market. In this case, servers have little incentive to compete on prices and each server adds a large mark-up over their own break-even price. As the competition parameter continues to decrease, servers lose their incentive to set prices lower than the market average. Because there is no central price coordinator to keep the average price low under very weak competition, the average market price may get dangerously high and decrease the market-size component of demand. Absence of demand due to high prices may lead to significant loss of profits for both the platform and the servers. We refer to this phenomenon as the *double-marginalization effect*.

The effect of a server's price on the average price is negligible, and therefore, drivers do not sufficiently internalize the effect of their decision on the market size. As platform attractiveness decreases, e.g. the parameter β increases, servers do not sufficiently lower their prices to make up for the smaller market. The server entry remains excessively high and the demand stays low. Effectively, the ratio of supply to demand monotonically increases in β. This highlights that server pricing is a risky policy. In extreme market conditions, whether it be market competition or platform attractiveness, server pricing may fail to retain a balance between supply and demand. In order for server pricing to generate coordination, servers' concerns for their own competitiveness should not overweight or underweight their concern for platform attractiveness.

4.3 Central Coordinator Pricing (Benchmark)

The average market price regulates both the market size and the competitiveness of servers. Due to its influence on both components of demand, the aforementioned price inefficiencies are realized through their effect on the average price. In order to understand the scale of the price inefficiencies, we define an "efficient average market price" as a reference point.

Assume that servers participate under a commission contract with $\phi = 1/2$ and a central price coordinator with perfect cost information sets the prices for all servers such that the servers' total profit is maximized. A central planner can consider the effect of a server's price on the rest of the market, so it can choose prices that are optimal for servers as a whole and not necessarily optimal at the individual level. Consider the central coordinator's problem:

$$\begin{aligned} \max_{p(c),\hat{c}} \quad & \int_0^{\hat{c}} \pi(c)\, dc = \int_0^{\hat{c}} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c))) \left(\frac{p(c)}{2} - c\right)\, dc \\ \text{s.t.} \quad & \text{Eq. (2).} \end{aligned} \tag{12}$$

Proposition 3. *If the market is "coordinated" by a central authority, the average market price is* $\overline{p} = \dfrac{2}{3\beta}$. *The average quantity served by a server is* $\overline{q} = \frac{1}{\hat{c}} \int_0^{\hat{c}} q(c)\, dc = \frac{1}{4}$.

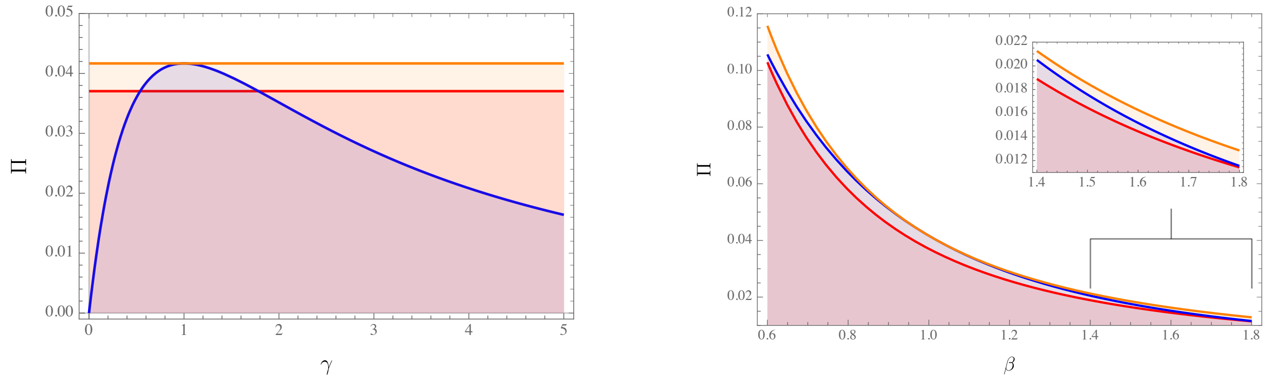
The proposition highlights that even under price heterogeneity, if the market is not susceptible to price inefficiencies, the average price should be insensitive to competition and be close to the central planner's optimal. By keeping the average quantity served per driver constant under all market conditions, the central planner ensures a balance between supply and demand. If this is achieved, we say the market is "coordinated". If the average price is not coordinated, then we can observe the scale of the price efficiencies through how much the average price is away from the optimal.

Comparing decentralized server pricing with this central planner, we find that the average prices are equivalent if $\gamma = \beta$. As γ increases or decreases, the ratio of average prices under the two policies monotonically diverge from each other. As γ increases, the servers gets concerned more with competitive effects than market size and decrease their price too much. Similarly, as γ decrease, the average price in the market gets too large. In that sense, the excessive movement of average price under server pricing triggers the price inefficiencies we consider in Section 4.2.

4.4 When Does the Platform Prefer Server Pricing?

We compare the platform's earning under both pricing policies in Figure 3. In order to eliminate trivial solutions from our results, e.g. cases where supply market is fully cleared, we limit our parameter space to $\beta \geq \frac{3}{4}$. Due to coordination inefficiencies observed under server pricing in the case of extreme market competition, the platform doesn't prefer server pricing if market competition is sufficiently weak or strong. When server pricing works well, it may generate higher profits than platform pricing. The ratio of earnings under server pricing to platform pricing is maximized for $\gamma = \beta$, at which server pricing leads to 12.5% higher earnings. For other cases, the ratio monotonically decreases as γ deviates away from β. When market competition is at its extremes, the ratio converges to 0.

The advantage of server pricing is its ability to attract large supply. By allowing servers to tailor their prices to their own costs, server pricing allows high cost servers to push up their margins to remain profitable at the expense of serving less demand. The downside of server pricing, as discussed, is its susceptibility to market effects. While platform pricing retains a fixed ratio of supply to demand, the ratio is monotonically increasing in β and decreasing in γ under server pricing. Self-interested servers over-react to changes in market competition and under-react to changes in platform's attractiveness. We summarize our results in Proposition 4.



(a) The platform's profits, Π, with respect to the market competition parameter, γ, for $\beta = 1$.

(b) Platform's profits, Π, with respect to the platform attractiveness parameter, β, for $\gamma = 1$.

Figure 3: Platform's profit under the three pricing policies: platform pricing (red), server pricing (blue).

Proposition 4. *(i) The ratio of earnings under two pricing policies is* $\frac{\Pi^{\mathcal{S}}}{\Pi^{\mathcal{P}}} = \frac{27\beta^2\gamma}{(2\beta+\gamma)^3}$. *The ratio is maximized at* $\gamma = \beta$ *and monotonically decreases towards 0 as* γ *deviates away from* β. *(ii) The ratio of server entry under two pricing policies is* $\frac{\hat{c}^{\mathcal{S}}}{\hat{c}^{\mathcal{P}}} = \frac{9\beta}{2(2\beta+\gamma)}$. *The ratio is increasing in* γ *and decreasing in* β. *(ii) Platform profits are higher under server pricing if and only if total server profit is also higher under server pricing.*

The second part of Proposition 4 highlights that when choosing the pricing policy, there is no incentive misalignment between the platform and the servers. The policy that maximizes platform's profit also maximizes server's profits. This observation might have policy implications. Although pricing control may appear restrictive due to its control over servers in the context of employee/contractor classification, this policy may also provide better compensation to the servers. In fact, under certain cases, platform pricing may be beneficial to all parties. When the competition in the market is sufficiently weak, platform pricing also decreases the prices in the market and increases customer's utility.

5 Optimal Contract Design

In this section, we characterize the optimal contract. The optimal contract grants the platform greater flexibility in setting prices that vary across the market and allows the platform to utilize more complex fee structures.

Let $p(c)$ be the price the platform assigns to server c and $f(c)$ be the fee collected. Under this mechanism, the platform also controls which servers participate. By the revelation principle, we can limit our focus to those mechanisms that are truth-inducing. If server c reports cost c_j, we denote the server's earning with $\pi(c, c_j)$. Conditional on participation, server c earns:

$$\pi(c, c) = (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))(p(c) - c) - f(c) \tag{13}$$

The incentive compatibility constraint implies:

$$\begin{aligned} \pi(c, c) \geq \pi(c, c_j) &= (1 - \beta\overline{p} + \gamma(\overline{p} - p(c_j))) \, (p(c_j) - c) - f(c_j) \\ &> (1 - \beta\overline{p} + \gamma(\overline{p} - p(c_j))) \, (p(c_j) - c_j) - f(c_j) = \pi(c_j, c_j). \end{aligned} \tag{14}$$

for all $c_j > c$. If we denote the highest cost that participates with $\hat{c}$, then this condition implies

$$\pi(\hat{c}, \hat{c}) = 0 \tag{15}$$

as otherwise the platform could uniformly increase the fee, $f(c)$, for all participating servers. Therefore, as an immediate consequence of truth-inducing mechanisms, we can focus on finding the highest-cost server that will participate without losing generality. The platform's optimal mechanism design problem is:

$$\begin{aligned} \max_{p(c), f(c), \hat{c}} \quad & \Pi = \int_0^{\hat{c}} f(c) \, dc \\ \text{s.t.} \quad & \pi(c_i, c_i) \geq \pi(c_i, c_j) \ , \forall c_i \in (0, \hat{c}), \forall c_j \in (0, \hat{c}) \\ & \pi(c_i, c_i) \geq 0 \ , \forall c_i \in (0, \hat{c}) \\ & \text{Eq. (2).} \end{aligned} \tag{16}$$

We solve for the platform's optimal choice of prices and fees.

Proposition 5. *There exists a unique solution to the platform's problem. The optimal contract exhibits the following characteristics: (i) The platform's choice of optimal price and fee are* $p(c) = \frac{2}{3\beta} - \frac{1}{6\gamma} + \frac{c\beta}{\gamma}$, $f(c) = c^2 \left(\frac{\beta(\gamma - 2\beta)}{2\gamma}\right) + \frac{2c}{3}\left(\frac{\beta}{\gamma} - 1\right) + \frac{1}{24}\left(\frac{5}{\beta} - \frac{2}{\gamma}\right)$. *(ii) The platform earns* $\Pi = \frac{1}{24\beta^2}$. *(iii) The equilibrium mass of participating servers and the average price are* $\hat{c} = \frac{1}{2\beta}$, $\overline{p} = \frac{2}{3\beta}$. *(iv) Total quantity served in the market is* $Q = \frac{1}{8\beta}$.

Similar to platform pricing, the optimal mechanism coordinates the market and mitigates destructive competitive effects. By keeping supply and demand insensitive to market competition, the optimal mechanism provides a robust balance to the market. While achieving robustness, the optimal mechanism also generates price heterogeneity, which helps increase platform profits above the other two pricing policies. This is attributed to two advantages. First, the platform can leverage cost information under the optimal mechanism. It can set higher prices for high-cost servers to promote server entry and lower prices for low-cost servers to keep the average price low. Second, the platform can utilize a complex fee structure. The platform's aggregate share of the total revenue is 1/2, which is similar to the two pricing policies we explored earlier; however, the fee collected from each server is not a fixed share of the revenue. Instead, the platform charges different commissions to different servers. The commission levels may be increasing or decreasing in cost depending on how the market's competition level compares to the platform's attractiveness. For example, in a competitive market, the platform charges lower commissions to high-cost servers to mitigate the competition effect which would otherwise push high-cost servers out of the market.

Although the optimal mechanism generates optimal returns for the platform, it may not be practical due to its reliance on truth-telling. In the next section, we seek an implementable optimal contract that doesn't require servers to explicitly announce their costs to the platform.

5.1 Commission Contract with Quantity Bonus/Surcharge

Focusing on the server pricing policy, we now consider a model where servers set their prices and the platform uses a quantity-based fee structure. Such structure has been shown to help the efficiency of decentralized supply chains (e.g. Monahan 1984, Weng 1995, Corbett and De Groote 2000), however, their use for price coordination has not been studied in the context of service platforms. The timeline of decisions under this policy is similar to server pricing in Figure 2. Rather than a fixed commission, the platform collects $f(q)$ for each unit served from a server that serves q customers. If $f(q)$ is decreasing in q, the platform essentially offers quantity bonuses. If $f(q)$ is increasing in q, the platform enforces quantity surcharges. Servers choose the quantities that maximize their profits. Server c faces the problem:

$$\begin{aligned} \max_{q(c)} \quad \pi(c) &= q(c)(p(c) - c) - q(c)f(q(c)) \\ &= q(c)\frac{1 + (\gamma - \beta)\overline{p} - c\gamma - q(c)}{\gamma} - q(c)f(q(c)). \end{aligned} \tag{17}$$

where the equality follows by plugging in $p(c)$ from Equation (1). The problem's solution satisfies

$$q(c) = \frac{1 + (\gamma - \beta)\overline{p} - c\gamma - \gamma f(q(c))}{2 + \gamma f'(q(c))}. \tag{18}$$

The highest cost participant is indifferent between participating and not participating:

$$\pi(\hat{c}) = 0. \tag{19}$$

Assuming Equations (2), (18), (19) are uniquely defined, platform's problem is:

$$\begin{aligned} \max_{f(q)} \quad \Pi^{\mathcal{Q}} \quad &= \int_0^{\hat{c}} q(c) f(q(c))\, dc \\ \text{s.t.} \quad & \text{Eq. } (2), (18), (19). \end{aligned} \tag{20}$$

Proposition 6. *There exists a unique solution to platform's problem. The optimal quantity-bonus policy exhibits the following characteristics: (i) Platform's optimal fee is* $f(q) = q\left(\frac{1}{2\beta} - \frac{1}{\gamma}\right) + \left(\frac{1}{6\beta} + \frac{1}{3\gamma}\right)$. *(ii) Platform's profits and other equilibrium market dynamics of the quantity-bonus model are equivalent to the optimal mechanism.*

Proposition 6 establishes that the platform can replicate the optimal mechanism without truth-telling by offering quantity bonuses. Due to price heterogeneity in the marketplace, quantity bonuses allow the platform to coordinate the market by influencing low-cost and high-cost servers' margins disproportionally.

In particular, as market competition weakens, the platform's optimal response is to increase its bonuses. By charging less fees to those servers attracting large demand, the platform expands low-cost servers' margins and shrinks high-cost servers' margins. This increases the variance of equilibrium prices set by the servers, which helps decrease the average market price without causing over-entry. As the market gets more competitive, the platform curbs the bonuses it offers. By reducing the servers' incentive to compete on prices, the platform eliminates the destructive competition observed under server pricing. When competition is too strong, $\gamma > 2\beta$, the platform's optimal strategy reverses. Rather than offering quantity bonuses, the platform imposes quantity surcharges to mitigate the excessive competition. This reduces server's incentive to compete in price and also expands the high-cost servers' margins to ensure they remain competitive. When constructing the optimal quantity bonus, the platform needs to take into consideration both the competitiveness of the market and the platform's attractiveness. By doing so, the platform can find a policy that eliminates the inefficiencies caused by servers' over-sensitivity to the market competitiveness and under-sensitivity to platform attractiveness.

Under this policy, total revenue generated in the system is $\frac{1}{12\beta^2}$. The platform and servers collect equal shares of the revenue. In the market, the average fee a server pays per customer is $\frac{1}{3\beta}$. As the platform becomes less attractive, the average fee paid decreases. The base fee, e.g. the marginal fee paid for serving the first customer, is $f(0) = \left(\frac{1}{6\beta} + \frac{1}{3\gamma}\right)$. The ratio of these rates is $\frac{2\gamma}{2\beta+\gamma}$, which is increasing in γ and decreasing in β. This means that when the platform attractiveness is low, the platform not only lowers the base fee, but also provides more expansive bonuses in an effort to lower the prices in the market. When the market is competitive, the platform lowers the base fee, but doesn't expand its bonuses. Instead, by curbing the bonuses it offers, the platform reduces the incentive of low-cost servers to compete in prices.

While useful under server pricing, quantity bonuses are redundant for platform pricing. Under uniform prices, all participating servers serve the same quantity. Although bonuses may influence a server's marginal utility gain from higher quantities, the server cannot act on this without influencing prices. In practice, service platforms may find other uses for quantity bonuses in practice. For example, platforms like Uber and Lyft offer quantity bonuses for driver retention. Our work proposes another motivation for use of quantity discounts in decentralized service platforms, by illustrating its role in price coordination.

6 Market Implications

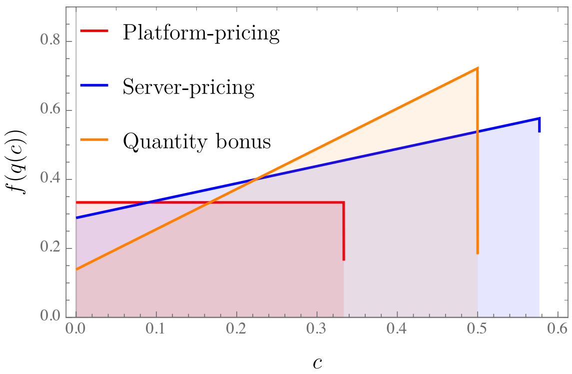
Platform pricing and quantity-bonus model are similar in their robustness to market competition. Although platform pricing is not optimal for the platform, it is consistently close to optimal. The earnings under platform pricing are approximately 11% lower than the quantity-bonus model everywhere. Despite not being optimal, its simple structure makes it attractive for the platform.

On the other hand, server pricing can generate higher profits under certain conditions. If $\gamma = \beta$, at which server pricing works best, server pricing replicates the optimal quantity-bonus model. Everywhere else, the quantity-bonus model generates strictly higher profits due to its ability to coordinate prices.

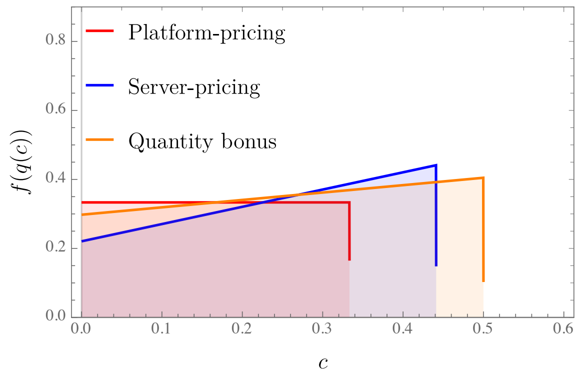
To understand the implications of pricing policies on the market, we focus on their effects on price formation. Under platform pricing, prices are fixed at $p(c) = \frac{2}{3\beta}$ for all γ values. Conversely, server pricing generates price heterogeneity, but the rate at which prices change with respect to server costs is constant at $\frac{\partial q(c)}{\partial c} = 1$. This behavior contributes to server pricing's coordination problem and explains why the average price is susceptible to market competition under this policy. When prices are linear in costs in the marketplace, keeping the median price fixed, increasing price heterogeneity decreases the average price.

As commission levels do not directly influence average prices, there are two control levers the platform can utilize to increase (decrease) the average price under strong (weak) competition: setting the prices or decreasing (increasing) the price heterogeneity in the market. Server pricing cannot control the average price in the market because it fails to achieve either of these objectives.

This weakness, however, can be remedied through quantity bonuses. In a quantity-bonus policy, the prices increase at a rate of $\frac{\partial q(c)}{\partial c} = \frac{\beta}{\gamma}$. Consequently, the price heterogeneity increases as market



(a) Fee paid per customer, $f(q(c))$, with respect to server cost, c, for $\gamma = 0.6$.



(b) Fee paid per customer, $f(q(c))$, with respect to server cost, c, for $\gamma = 1.4$.

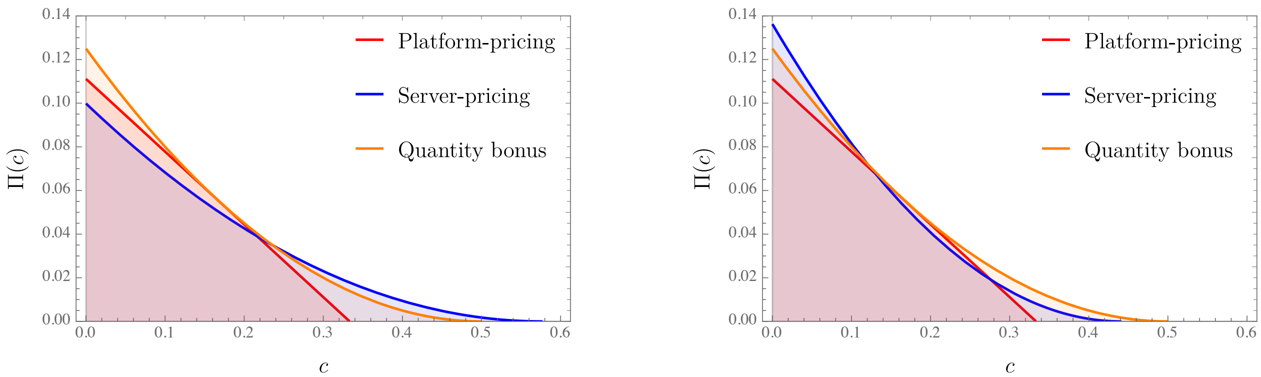
Figure 4: Fee paid per customer by servers to the platform under the three pricing policies for $\beta = 1$.

competition weakens or platform becomes less attractive. If market competition is significantly weak, $\gamma < \beta/4$, low-cost servers may pay money to the customers to serve them, meanwhile high cost servers charge premium prices. As competition intensifies, the price heterogeneity shrinks. This exerts an upward pressure on the average price and mitigates the effect of price competition on the average price. The existence of small price heterogeneity still allows platform to attract a large supply.

We plot the total fee each server pays in the market for two cases: weak competition ($\gamma = 0.8\beta$) and strong competition ($\gamma = 1.2)\beta$. For simplicity, we normalize β to 1. The fees are in Figure 4. Under weak competition, the quantity-bonus model subsidizes low-cost servers. When low-cost servers have very low prices, the size of the market expands and allows the platform to extract more fees from the high-cost servers who charge premium prices. Similar dynamics hold under server pricing, albeit at a smaller scale. The fee servers pay under server pricing is not sufficiently concave in costs, so the platform does not provide adequate incentive to low-cost servers to keep their prices very low. As the market gets more competitive, subsidization vanishes under both models.

Under the quantity-bonus model, the fees become less concave. This implies that high-cost servers pay less fees and secure healthier margins, which is the result of platform's effort to prevent destructive competition. Server pricing, on the other hand, exhibits a stronger concavity, which does a worse job at mitigating competitive behavior. This rigidness in the fee structure is one of the main mechanics leading to server pricing's susceptibility to price coordination failures.

For the same two cases, we also plot servers' profits in Figure 5. Under platform pricing, servers' net profits are linear decreasing in costs. The quantity-bonus model utilizes quantity bonuses to generate convexity in server earnings, which helps high cost servers profitably participate. Although the server earnings under server pricing are also convex, unlike the other policies, they are sensitive to market competition. In a competitive market, server earnings drop very fast and server supply becomes very limited. Under weak competition, server pricing attracts a larger supply, however, this comes at the expense of low demand. We summarize our results in Proposition 7.



(a) Servers' profits, $\pi(c)$, with respect to server cost, c, for $\gamma = 0.6$.

(b) Servers profit's, $\pi(c)$, with respect to server cost, c, for $\gamma = 1.4$.

Figure 5: Servers' profits under the three pricing policies for $\beta = 1$.

Proposition 7. *(i) Under quantity bonus model, platform's profit is 10% higher than platform-pricing. The ratio of earnings under quantity bonus and server pricing is* $\frac{\Pi^{\mathcal{Q}}}{\Pi^{\mathcal{S}}} = \frac{(2\beta+\gamma)^3}{24\beta^2\gamma}$*. The ratio is minimized at* $\gamma = \beta$ *and is equal to 1. As* γ *deviates from* β*, the ratio monotonically increases. (ii) Quantity bonus model and platform pricing lead to same average prices. The average price is strictly lower than both policies under server pricing if* $\gamma > \beta$ *and is higher otherwise. (iii) Quantity bonus model leads to 50% higher server entry than platform pricing. Server entry under server pricing is higher than platform pricing if* $\gamma < \frac{5\beta}{2}$ *and is higher than quantity bonus model if* $\gamma < \beta$*.*

Our observations underline quantity bonus' use as a tool that helps platform influence servers' competitive behavior. When market competition is too weak, quantity bonuses make low prices attractive to low cost servers and helps retain a low average price. When competition is strong, quantity surcharges eliminate the attractiveness of competitive behavior and ensures that the average price in the market is not destructively low. By promoting coordination, quantity bonuses help platform retain a balance in the market.

Server and Customer Welfare

Another relevant question we look at is whether servers prefer one pricing policy over the others. Interestingly, under all pricing policies, servers retain half of the revenues, which corresponds to 1/3 of the profits generated in the system. This shows that the platform and the servers have aligned incentives in choosing the best pricing policy.

To investigate the customers' utility, we can use the average market price and the quantity served in the market as a proxy. The quantity-bonus model leads to the same average price and higher quantity served compared to platform pricing. In that sense, customers are always better off under quantity-bonus model than platform pricing. Server pricing leads to lower average price than quantity-bonus for $\gamma > \beta$ and higher quantity served for $\beta < \gamma < \beta$. So, when the market is slightly competitive server pricing leads to higher customer utility. However, if the market gets too competitive the customer utility goes down. This is because when the competition is destructive server entry gets too low, which hurts quantity served. Comparing platform- and server pricing, the quantity served is higher under server pricing for $0.75\beta < \gamma < 5.38\beta$. So, customer utility is higher under server pricing for intermediate values of γ. If market competition is too weak or too strong, server pricing leads to lower customer utility. We note these results in Proposition 8.

Proposition 8. *(i) Under all pricing policies, the servers retain half of the revenue generated and their total profits is equal to half of platform's profits. The policy that maximizes platform's profits also maximize servers' profits. (ii) The customer utility under quantity bonus is higher than platform pricing. Server pricing can lead to higher customer utility than both policies, however, this comes at the expense of low lower profits for the platform.*

7 Remarks

This papers deals with a fundamental question for service platforms: When servers have heterogeneous costs and cost information is private, who should set the prices in service platforms? If platform assumes pricing role as the central authority, it can achieve coordination among servers. However, the platform has to act with limited cost information. To utilize private information, the platform can allow servers to set their own prices. Such an approach can be risky under a simple commission contract. As servers are self-interested, the resulting prices may be too low or too high and non-optimal for the platform. In order to mitigate these inefficiencies, we find it optimal to offer quantity bonuses to server. Our results show that when correct measure are in place to ensure coordination in the system, decentralized pricing functions efficiently. Compared to platform pricing, it can promote server entry and increase both the platform's and servers' profits.

Our results have practical implications. For a new startup that is established in a growing market, server pricing may appear advantageous due to its supply benefits. Supply-constrained platforms may be drawn to the prospect of additional server providers signing up for the service in return for greater operational freedom. We are recently seeing this trend with startups that use blockchain technology to decentralize the pricing control without proposing sufficient provisions to guarantee coordination. Our results show that server pricing's supply benefits can be excessive and accompanied by price inefficiencies, which lead to sub-optimal earnings. In order to mitigate these issues, we find it important for the platforms to design appropriate measures to ensure price coordination.

Our research also promotes an alternative use of quantity bonuses for service platforms. Although quantity bonuses are used by platforms in practice, their considerations are generally limited to supply retention. We find that quantity bonuses can also help the platform generate price coordination when pricing is decentralized. To design a successful quantity bonus structure, the platform has to take into consideration both its own attractiveness to the customers and also servers' competitiveness. By doing so, platform can mitigate pricing inefficiencies associated with decentralized pricing.

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Appendix (TO BE MODIFIED)

Proof of Proposition 1. Platform's problem is:

$$\begin{aligned}\max_{p,\phi} \quad \Pi &= \phi p \int_0^{\hat{c}} (1-p)\, dc \\ &= \phi p \int_0^{(1-\phi)p} (1-p)\, dc \\ &= \phi(1-\phi)p^2(1-p)\end{aligned} \tag{21}$$

The maximizer is concave in ϕ and quasi-concave in p. Therefore, platform's solution is unique.

1. The FOC gives:

$$\begin{aligned}\frac{\partial}{\partial p}\left(\phi(1-\phi)p^2(1-p)\right) = 0 &\implies p(c) = \frac{2}{3}, \\ \frac{\partial}{\partial \phi}\left(\phi(1-\phi)p^2(1-p)\right) = 0 &\implies \phi = \frac{1}{2}.\end{aligned} \tag{22}$$

$$\Pi = \phi(1-\phi)p^2(1-p) = \frac{1}{27}. \tag{23}$$

2.

$$\begin{aligned}\hat{c} = (1-\phi)p = \frac{1}{3}, \\ \bar{p} = p = \frac{2}{3}.\end{aligned} \tag{24}$$

3.

$$\begin{aligned}Q &= \int_0^{\hat{c}} (1-p)\, dc \\ &= (1-\phi)p(1-p) = \frac{1}{9}.\end{aligned} \tag{25}$$

□

Proof of Proposition 2. The average market price is

$$\overline{p} = \frac{\int_0^{\hat{c}} q(c)p(c)\ dc}{\int_0^{\hat{c}} q(c)\ dc}$$

$$= \frac{\int_0^{\hat{c}} \frac{(1+(\gamma-1)\overline{p})^2}{4\gamma} - \frac{c^2\gamma}{4(1-\phi)^2}\ dc}{\int_0^{\hat{c}} \frac{1}{2}\left(1+(\gamma-1)\overline{p} - \frac{c\gamma}{1-\phi}\right)\ dc} \tag{26}$$

$$= \frac{4(\phi-1)\left(\frac{\hat{c}((\gamma-1)\overline{p}+1)^2}{4\gamma} - \frac{\gamma\hat{c}^3}{12(\phi-1)^2}\right)}{\hat{c}(\gamma\hat{c}+2(\gamma-1)\overline{p}(\phi-1)+2\phi-2)}.$$

$$= \frac{2((\gamma-1)\overline{p}+1)}{3\gamma}.$$

Solving for $\overline{p}$ gives:

$$\overline{p} = \frac{2}{2+\gamma}. \tag{27}$$

Platform's problem is:

$$\begin{aligned} \max_{\phi} \quad \Pi &= \phi\overline{p}\int_0^{\hat{c}} (1-\overline{p}+\gamma(\overline{p}-p(c)))\ dc \\ &= \frac{1}{4}\hat{c}\overline{p}\phi\left(\frac{\gamma\hat{c}}{\phi-1} + 2(\gamma-1)\overline{p}+2\right) \\ &= \frac{\overline{p}(1-\phi)\phi((\gamma-1)\overline{p}+1)^2}{4\gamma} \\ &= \frac{9\gamma(1-\phi)\phi}{2(\gamma+2)^3} \end{aligned} \tag{28}$$

The maximizer is concave in ϕ. Therefore, platform's solution is unique.

1. The FOC gives:

$$\begin{gathered} \frac{\partial}{\partial\phi}\left(\frac{9\gamma(1-\phi)\phi}{2(\gamma+2)^3}\right) = 0 \implies \phi = \frac{1}{2}, \\ p(c) = \frac{1}{2\gamma}\left(1+(\gamma-1)\overline{p}+\frac{c\gamma}{1-\phi}\right) = \frac{3}{2(2+\gamma)} + c. \end{gathered} \tag{29}$$

$$\Pi = \frac{9\gamma(1-\phi)\phi}{2(\gamma+2)^3} = \frac{9}{8}\left(\frac{\gamma}{(2+\gamma)^3}\right). \tag{30}$$

2.

$$\hat{c} = \frac{(1-\phi)(1+(\gamma-1)\overline{p})}{\gamma} = \frac{3}{2(2+\gamma)},$$
$$\overline{p} = \frac{2}{2+\gamma}. \tag{31}$$

3.

$$\begin{aligned} Q &= \int_0^{\hat{c}} q(c)\, dc \\ &= \int_0^{\hat{c}} (1-\overline{p}+\gamma(\overline{p}-p(c)))\ dc \\ &= \frac{9}{8}\left(\frac{\gamma}{(2+\gamma)^2}\right) \end{aligned} \tag{32}$$

□

Proof of Proposition 4. Will add after finalizing the proposition

□

Proof of Proposition 5. By revelation principle, it is sufficient to focus only on those mechanisms that are truth-inducing. Each server announces a type, c_i'. Given the reported cost, the mechanism determines whether the server participates in the market, and if so, the server's price, $p(c_i')$, and the fee platform collects from the server, $f(c_i')$. Let $p_i = p(c_i')$ and $f_i = f(c_i')$. In the optimal mechanism each server has an incentive to report their type truthfully and earn non-negative profit. If a server with cost c_i reports a cost of c_i', the server's contribution to platform is

$$\begin{aligned} R_i &= q(c_i')p_i \\ &= (1-\overline{p}+\gamma(\overline{p}-p_i))\, p_i. \end{aligned} \tag{33}$$

and the server earns

$$\begin{aligned} \pi(c_i, c_i') &= R_i - c_i q(c_i') - f_i \\ &= (1-\overline{p}+\gamma(\overline{p}-p_i))\,(p_i-c_i) - f_i. \end{aligned} \tag{34}$$

Let

$$u(c_i, c_i') = (1-\overline{p}+\gamma(\overline{p}-p_i))\,(p_i-c_i). \tag{35}$$

Then, server's net earning is

$$\pi(c_i, c_i') = u(c_i, c_i') - f_i. \tag{36}$$

Notice that marginal utility from higher p_i is increasing with cost c_i. Specifically,

$$\frac{\partial}{\partial c_i}\frac{\partial \pi(c_i,c_i')}{\partial p_i(c_i')} = \frac{\partial}{\partial c_i}\left(\gamma(c_i+\overline{p}-2p_i)-\overline{p}+1\right) = \gamma > 0. \tag{37}$$

So, the single-crossing condition is satisfied for the problem.

By IR, we have

$$\pi(c_i,c_i') = u(c_i,c_i') - f_i \geq 0, \quad ,\forall i \in [0,n] \tag{38}$$

By IC, we have

$$\pi(c_i,c_i) \geq \pi(c_i,c_j) \quad ,\forall i,j \in [0,n] \tag{39}$$

Since the server i can report his cost as c_j, by IC we have

$$\begin{aligned}\pi(c_i,c_i) \geq \pi(c_i,c_j) &= \left(1-\overline{p}+\gamma(\overline{p}-p_j)\right)(p_j-c_i)-f_j \\ &> \left(1-\overline{p}+\gamma(\overline{p}-p_j)\right)(p_j-c_j)-f_j = \pi(c_j,c_j).\end{aligned} \tag{40}$$

for all $c_j > c_i$. If we denote the highest cost that participates with $\hat{c}$, then we also can say

$$\pi(\hat{c},\hat{c}) = 0 \tag{41}$$

as otherwise platform can uniformly increase the fee, f_i, for all participating servers.

The utility a type c_i cost server gets from reporting a cost of c_j is alternatively formulated as

$$\pi(c_i,c_j) = \pi(c_j,c_j) - u(c_j,c_j) + u(c_i,c_j) \tag{42}$$

Therefore, the pair of inequalities IC constraints imposes for servers with costs c_i and c_j are:

$$\pi(c_i,c_i) \geq \pi(c_j,c_j) - u(c_j,c_j) + u(c_i,c_j), \tag{43}$$

$$\pi(c_j,c_j) \geq \pi(c_i,c_i) - u(c_i,c_i) + u(c_j,c_i). \tag{44}$$

These inequalities can be combined as

$$\begin{aligned}&u(c_j,c_i)-u(c_i,c_i) \leq \pi(c_j,c_j)-\pi(c_i,c_i) \leq u(c_j,c_j)-u(c_i,c_j) \\ \iff &\int_{c_i}^{c_j}\frac{\partial u(c_k,c_i)}{\partial c_k}dc_k \leq \pi(c_j,c_j)-\pi(c_i,c_i) \leq \int_{c_i}^{c_j}\frac{\partial u(c_k,c_j)}{\partial c_k}dc_k\end{aligned} \tag{45}$$

where $\frac{\partial u(c_k,c_j)}{\partial c_k}$ is the partial derivative of u with respect to its first argument evaluated at the point (c_k,c_j).

This implies

$$0 \leq \int_{c_i}^{c_j} \frac{\partial u(c_k, c_j)}{\partial c_k} - \int_{c_i}^{c_j} \frac{\partial u(c_k, c_i)}{\partial c_k}$$

$$= \int_{c_i}^{c_j} \int_{p(c_i)}^{p(c_j)} \frac{\partial^2 u(c_k, c_z)}{\partial c_k \partial p(c_z)} dp(c_z) dc_k \tag{46}$$

Due to single crossing property the second derivative of u is non-negative. Therefore the expression above implies $p(c_j) \geq p(c_i)$ for all $c_j > c_i$. If platform sets $p(c_j) = p(c_i)$, then

$$u(c_i, c_j) = u(c_i, c_i) \text{ and } u(c_j, c_i) = u(c_j, c_j). \tag{47}$$

which implies

$$\pi(c_j, c_j) - \pi(c_i, c_i) = u(c_j, c_j) - u(c_i, c_i) \iff f_i = f_j \tag{48}$$

By fixing one end point and letting the other converge towards it, we can also infer that $u(c_k, c_k)$ is continuous and the derivative is

$$\frac{d\pi(c_i, c_i)}{dc_i} = \frac{\partial u(c_i, c_i)}{\partial c_i} \tag{49}$$

where $\frac{d\pi(c_i,c_i)}{dc_i}$ is the total derivative of π with respect to c_i evaluated at (c_i, c_i) and $\frac{\partial u(c_i,c_i)}{\partial c_i}$ is the partial derivative of u with respect to its first argument evaluated at the point (c_i, c_i).

Integration leads to

$$\pi(c_i, c_i) + \int_{c_i}^{c_j} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k = \pi(c_j, c_j). \tag{50}$$

Setting $c_j = \hat{c}$, the equation simplifies to

$$\pi(c_i, c_i) = -\int_{c_i}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k. \tag{51}$$

which is equivalent to

$$f_i = u(c_i, c_i) + \int_{c_i}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k. \tag{52}$$

Notice that the monotonicity of $p(c_i')$ and Equation (52) are necessary conditions implied by IC. We also would like to show that they are sufficient conditions. To prove this, assume Equation (52) holds and consider the utility server with cost c_i gets by reporting cost $c_j \neq c_i$. If $c_j > c_i$, we have

$$\begin{aligned}
\pi(c_i, c_j) &= u(c_i, c_j) - f_j \\
&= u(c_i, c_j) - u(c_j, c_j) - \int_{c_i}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k + \int_{c_i}^{c_j} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k \\
&= - \int_{c_i}^{c_j} \frac{\partial u(c_k, c_j)}{\partial c_k} dc_k - \int_{c_i}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k + \int_{c_i}^{c_j} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k \\
&= - \int_{c_i}^{c_j} \left(\frac{\partial u(c_k, c_j)}{\partial c_k} - \int_{c_i}^{c_j} \frac{\partial u(c_k, c_k)}{\partial c_k} \right) dc_k - \int_{c_i}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k \\
&= - \int_{c_i}^{c_j} \int_{p(c_k)}^{p(c_j)} \frac{\partial^2 u(c_k, c_z)}{\partial c_k \partial p(c_z)} dp(c_z) dc_k + \pi(c_i, c_i) \leq \pi(c_i, c_i)
\end{aligned} \tag{53}$$

which follows because single crossing property implies

$$\int_{c_i}^{c_j} \int_{p(c_k)}^{p(c_j)} \frac{\partial^2 u(c_k, c_z)}{\partial c_k \partial p(c_z)} dp(c_z) dc_k \geq 0 \tag{54}$$

whenever $p(c_k) \leq p(c_j)$. Therefore IC constraint is satisfied for $c_j > c_i$. If $c_j < c_i$, we have

$$\begin{aligned}
\pi(c_i, c_j) &= u(c_i, c_j) - f_j \\
&= u(c_i, c_j) - u(c_j, c_j) - \int_{c_j}^{c_i} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k - \int_{c_i}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k \\
&= \int_{c_j}^{c_i} \frac{\partial u(c_k, c_j)}{\partial c_k} dc_k - \int_{c_j}^{c_i} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k - \int_{c_i}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k \\
&= \int_{c_j}^{c_i} \left(\frac{\partial u(c_k, c_j)}{\partial c_k} - \frac{\partial u(c_k, c_k)}{\partial c_k} \right) dc_k - \int_{c_i}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k \\
&= \int_{c_j}^{c_i} \int_{p(c_k)}^{p(c_j)} \frac{\partial^2 u(c_k, c_z)}{\partial c_k \partial p(c_z)} dp(c_z) dc_k + \pi(c_i, c_i) \leq \pi(c_i, c_i)
\end{aligned} \tag{55}$$

which follows because single crossing property implies

$$\int_{c_j}^{c_i} \int_{p(c_k)}^{p(c_j)} \frac{\partial^2 u(c_k, c_z)}{\partial c_k \partial p(c_z)} dp(c_z) dc_k \leq 0 \tag{56}$$

whenever $p(c_k) \geq p(c_j)$. Therefore two necessary and sufficient conditions for IC constraint is non-decreasing $p(c_i')$ and Equation (52).

The platform's IC fee simplifies to

$$
\begin{aligned}
f_i &= u(c_i, c_i) + \int\limits_{c_i}^{\hat{c}} (-q(c_k)) dc_k \\
&= \left(1 - \overline{p} + \gamma(\overline{p} - p_i)\right)(p_i - c_i) - \int\limits_{c_i}^{\hat{c}} \left(1 - \overline{p} + \gamma(\overline{p} - p_k)\right) dc_k
\end{aligned}
\tag{57}
$$

where $p_k = p(c_k)$.

The platform's problem is to choose $p(c_i')$, $f(c_i')$ and $\hat{c}$ to maximize the total earnings subject to IR and IC constraints.

$$
\begin{aligned}
\max_{p(c_i), f(c_i), \hat{c}} \quad & \int\limits_0^{\hat{c}} f_i dc_i \\
\text{s.t.} \quad & \pi_i \geq 0, \forall i \\
& p'(c_i') \geq 0, \forall c_i' \\
& f_i = u(c_i, c_i) + \int\limits_{c_i}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k. \\
= \max_{p(c_i), \hat{c}} \quad & \int\limits_0^{\hat{c}} \left(u(c_i, c_i) + \int\limits_{c_i}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k \right) dc_i \\
\text{s.t.} \quad & \pi_i \geq 0, \forall i \\
& p'(c_i') \geq 0, \forall c_i'
\end{aligned}
\tag{58}
$$

Notice that $u(c_i, c_i)$ is equivalent to the welfare generated by server i. Therefore the platform's optimal mechanism will not be welfare-maximizing.

Using integration by parts we have

$$
\begin{aligned}
\int\limits_0^{\hat{c}} \left(\int\limits_{c_i}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k \right) dc_i &= \left[\left(\int\limits_{c_i}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} dc_k \right) c_i \right]_0^{\hat{c}} - \int\limits_0^{\hat{c}} \left(-\frac{\partial u(c_i, c_i)}{\partial c_i} \right) c_i dc_i \\
&= \int\limits_0^{\hat{c}} \frac{\partial u(c_i, c_i)}{\partial c_i} c_i dc_i
\end{aligned}
\tag{59}
$$

Then, platform's problem converts to

$$
\begin{aligned}
&\max_{p(c_i),\hat{c}} && \int_0^{\hat{c}} \left(u(c_i, c_i) + \frac{\partial u(c_i, c_i)}{\partial c_i} c_i \right) dc_i \\
&\text{s.t.} && \pi_i \geq 0, \forall i \\
& && p'(c_i') \geq 0, \forall c_i' \\
&= \max_{p(c_i),\hat{c}} && \int_0^{\hat{c}} \Big((1 - \overline{p} + \gamma(\overline{p} - p_i))(p_i - c_i) - (1 - \overline{p} + \gamma(\overline{p} - p_i))c_i \Big) dc_i \\
&\text{s.t.} && \pi_i \geq 0, \forall i \\
& && p'(c_i') \geq 0, \forall c_i' \\
&= \max_{p(c_i),\hat{c}} && \int_0^{\hat{c}} (1 - \overline{p} + \gamma(\overline{p} - p_i))(p_i - 2c_i) dc_i \\
&\text{s.t.} && \pi_i \geq 0, \forall i \\
& && p'(c_i') \geq 0, \forall c_i'
\end{aligned}
\tag{60}
$$

Let us assume that $\hat{c}$ and $\overline{p}$ are fixed and consider the platform's problem:

$$
\begin{aligned}
&\max_{p(c)} && \int_0^{\hat{c}} (1 - \overline{p} + \gamma(\overline{p} - p(c)))(p(c) - 2c)\ dc \\
&\text{s.t.} && \frac{\int_0^{\hat{c}} (1 - \overline{p} + \gamma(\overline{p} - p(c)))p(c)\ dc}{\int_0^{\hat{c}} (1 - \overline{p} + \gamma(\overline{p} - p(c)))\ dc} = \overline{p} \\
& && p'(c) \geq 0 \\
&= \max_{p(c)} && \int_0^{\hat{c}} (1 - \overline{p} + \gamma(\overline{p} - p(c)))(p(c) - 2c)\ dc \\
&\text{s.t.} && \int_0^{\hat{c}} (1 - \overline{p} + \gamma(\overline{p} - p(c)))p(c)\ dc = \overline{p} \left(\int_0^{\hat{c}} (1 - \overline{p} + \gamma(\overline{p} - p(c)))\ dc \right) \\
& && p'(c) \geq 0
\end{aligned}
\tag{61}
$$

Notice that the total demand served is

$$
\int_0^{\hat{c}} (1 - \overline{p} + \gamma(\overline{p} - p(c)))\ dc = \hat{c} \left(1 + (\gamma - 1)\overline{p} - \gamma \frac{1}{\hat{c}} \int_0^{\hat{c}} p(c)\ dc \right), \tag{62}
$$

so the problem transforms to

$$\begin{aligned}
\max_{p(c)} \quad & \int_0^{\hat{c}} (1-\overline{p}+\gamma(\overline{p}-p(c)))(p(c)-2c)\ dc \\
\text{s.t.} \quad & \int_0^{\hat{c}} (1-\overline{p}+\gamma(\overline{p}-p(c)))p(c)\ dc = \overline{p}\hat{c}\left(1+(\gamma-1)\overline{p}-\gamma\frac{1}{\hat{c}}\int_0^{\hat{c}} p(c)\ dc\right) \\
& p'(c) \geq 0
\end{aligned} \tag{63}$$

Let

$$p(c) = \frac{1-\overline{p}+\gamma\overline{p}+2c\gamma}{2\gamma} + \delta(c). \tag{64}$$

where $\delta(c)$ be the platform's deviation from server-optimal price. Since $\overline{p}$ and $\hat{c}$ are fixed, we can re-write platform's problem as a function of $\delta(c)$. After we plug in the values of $p(c)$ and simplify the expressions the problem becomes:

$$\begin{aligned}
\max_{\delta(c)} \quad & \int_0^{\hat{c}} \frac{(-2c\gamma+(\gamma-1)\overline{p}+1)^2}{4\gamma}\ dc - \gamma\int_0^{\hat{c}} \delta(c)^2\ dc \\
\text{s.t.} \quad & \int_0^{\hat{c}} \left(\gamma\delta(c)^2+2c\gamma\delta(c)-\gamma\overline{p}\delta(c)\right)\ dc = \frac{\hat{c}(\overline{p}-1)^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(4\hat{c}^2-6\hat{c}\overline{p}+3\overline{p}^2\right) \\
& \delta'(c) \geq -1
\end{aligned} \tag{65}$$

Notice that since $\overline{p}$ and $\hat{c}$ are fixed, the problem is equivalent to

$$\begin{aligned}
\max_{\delta(c)} \quad & -\gamma\int_0^{\hat{c}} \delta(c)^2\ dc \\
\text{s.t.} \quad & \int_0^{\hat{c}} \left(\gamma\delta(c)^2+2c\gamma\delta(c)-\gamma\overline{p}\delta(c)\right)\ dc = \frac{\hat{c}(\overline{p}-1)^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(4\hat{c}^2-6\hat{c}\overline{p}+3\overline{p}^2\right) \\
& \delta'(c) \geq -1 \\
= \min_{\delta(c)} \quad & \int_0^{\hat{c}} \delta(c)^2\ dc \\
\text{s.t.} \quad & \int_0^{\hat{c}} \left(\gamma\delta(c)^2+2c\gamma\delta(c)-\gamma\overline{p}\delta(c)\right)\ dc = \frac{\hat{c}(\overline{p}-1)^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(4\hat{c}^2-6\hat{c}\overline{p}+3\overline{p}^2\right) \\
& \delta'(c) \geq -1
\end{aligned} \tag{66}$$

Let us solve the relaxed problem without the inequality constraint. We can use calculus of variations to solve this problem. The Lagrangian is

$$L = \int_0^{\hat{c}} \delta(c)^2\ dc + \lambda\left(\frac{\hat{c}(\overline{p}-1)^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(4\hat{c}^2-6\hat{c}\overline{p}+3\overline{p}^2\right) - \int_0^{\hat{c}} \left(\gamma\delta(c)^2+2c\gamma\delta(c)-\gamma\overline{p}\delta(c)\right)\ dc\right) \tag{67}$$

The Lagrange-Euler equation is

$$\frac{\partial}{\partial\delta(c)}\Big(\delta(c)^2\ dc - \lambda\left(\gamma\delta(c)^2+2c\gamma\delta(c)-\gamma\overline{p}\delta(c)\right)\Big) = 0,\ \forall c \tag{68}$$

This gives

$$\delta(c) = \frac{\gamma\lambda\overline{p} - 2c\gamma\lambda}{2(\gamma\lambda - 1)}, \ \forall c \tag{69}$$

We plug this into the constraint to find λ. We have

$$\int_0^{\hat{c}} \left(\gamma \left(\frac{\gamma\lambda\overline{p} - 2c\gamma\lambda}{2(\gamma\lambda - 1)} \right)^2 + 2c\gamma \left(\frac{\gamma\lambda\overline{p} - 2c\gamma\lambda}{2(\gamma\lambda - 1)} \right) - \gamma\overline{p} \left(\frac{\gamma\lambda\overline{p} - 2c\gamma\lambda}{2(\gamma\lambda - 1)} \right) \right) dc = \frac{\hat{c}(\overline{p} - 1)^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2\right) \tag{70}$$

We get two solutions

$$\lambda = \frac{1}{\gamma} \pm \frac{\sqrt{\frac{4\hat{c}^2}{3} - 2\hat{c}\overline{p} + \overline{p}^2}}{\overline{p} - 1}, \tag{71}$$

We can plug in the λ to find the optimal $\delta(c)$ and therefore the optimal $p(c)$.

If $\lambda = \frac{1}{\gamma} + \frac{\sqrt{\frac{4\hat{c}^2}{3} - 2\hat{c}\overline{p} + \overline{p}^2}}{\overline{p} - 1}$, we get

$$p(c) = \frac{-6c(1 - \overline{p}) - \overline{p}\left(\sqrt{3}(1 - 2\gamma)\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2} + 3\overline{p}\right) + \sqrt{3}\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2} + 3\overline{p}}{2\sqrt{3}\gamma\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2}} \tag{72}$$

Notice that this solution is not feasible, since the price is decreasing in cost, c. Let us look at the other solution. If $\lambda = \frac{1}{\gamma} - \frac{\sqrt{\frac{4\hat{c}^2}{3} - 2\hat{c}\overline{p} + \overline{p}^2}}{\overline{p} + 1}$, we get

$$p(c) = \frac{6c(1 - \overline{p}) + \overline{p}\left(\sqrt{3}(2\gamma - 1)\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2} + 3\overline{p}\right) + \sqrt{3}\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2} - 3\overline{p}}{2\sqrt{3}\gamma\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2}} \tag{73}$$

The price is increasing in c, so the solution is feasible. As $p(c)$ is a linear function, we can conclude that in the optimal mechanism prices will be linear. We now solve the optimal mechanism problem using the linearity assumption.

Let $p(c) = a_0 + a_1 c$. The platform's problem is

$$\begin{aligned} \max_{a_0, a_1, \hat{c}} \quad & \int_0^{\hat{c}} (1 - \overline{p} + \gamma(\overline{p} - (a_0 + a_1 c)))(a_0 + a_1 c - 2c) \, dc \\ \text{s.t.} \quad & 1 - \overline{p} + \gamma(\overline{p} - (a_0 + a_1 c)) \geq 0 \\ & a_1 \geq 0 \end{aligned} \tag{74}$$

where

$$\overline{p} = \frac{\int_0^{\hat{c}} (1 - \overline{p} + \gamma(\overline{p} - (a_0 + a_1 c)))(a_0 + a_1 c) \, dc}{\int_0^{\hat{c}} (1 - \overline{p} + \gamma(\overline{p} - (a_0 + a_1 c))) \, dc} \tag{75}$$

The optimal solution is

$$a_0 = \frac{2}{3} - \frac{1}{6\gamma},$$
$$a_1 = \frac{1}{2\gamma}, \tag{76}$$
$$\hat{c} = \frac{1}{2}.$$

1.

$$p(c) = a_0 + a_1 c = \frac{2}{3} - \frac{1}{6\gamma} + \frac{c}{\gamma},$$
$$f(c) = (1 - \overline{p} + \gamma(\overline{p} - p(c)))\,(p(c) - c) - \int\limits_c^{\hat{c}} (1 - \overline{p} + \gamma(\overline{p} - p(c_i)))\,dc_i \tag{77}$$
$$= \frac{5}{12} - \frac{1}{6\gamma} + \frac{2-\gamma}{2\gamma}c.$$

The platform earns

$$\Pi = \int\limits_0^{\hat{c}} (1 - \overline{p} + \gamma(\overline{p} - (a_0 + a_1 c)))(a_0 + a_1 c - 2c)\;dc = \frac{1}{24}. \tag{78}$$

2.

$$\hat{c} = \frac{1}{2},$$
$$\overline{p} = \frac{\int\limits_0^{\hat{c}} (1 - \overline{p} + \gamma(\overline{p} - (a_0 + a_1 c)))(a_0 + a_1 c)\;dc}{\int\limits_0^{\hat{c}} (1 - \overline{p} + \gamma(\overline{p} - (a_0 + a_1 c)))\;dc} = \frac{2}{3}. \tag{79}$$

3.

$$Q = \int\limits_0^{\hat{c}} (1 - \overline{p} + \gamma(\overline{p} - (a_0 + a_1 c)))\;dc = \frac{1}{8}. \tag{80}$$

□