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The distribution of long-term firm performance

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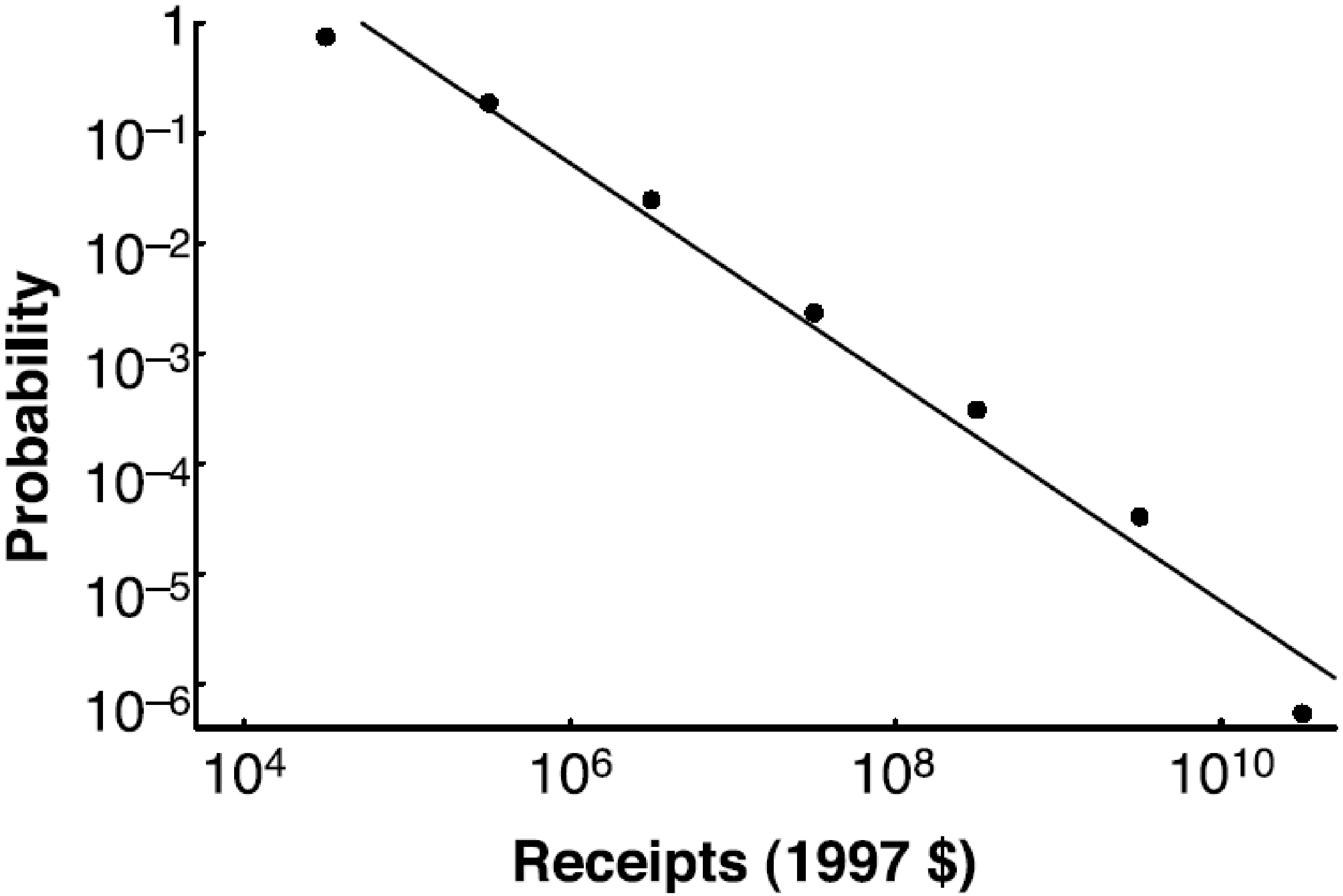
Firm sizes: close to Zipf's law (or log-normal)



Source: Axtell (2001) "Zipf Distribution of U.S. Firm Sizes", Science.

Gibrat's law: proportional growth rates

- Change in size X_t proportional to itself:

$$\Delta X_t = X_t \, \Delta g_t$$

- Growth rate has a normal distribution

- In continuous time x_t = log(X_t) follows a random walk:

$$\mathrm{d}x_t = \mu \, \mathrm{d}t + \sigma \, \mathrm{d}z_t$$

- Leads to a log-normal distribution of X_t

But what about profit?

Long-term economic profit

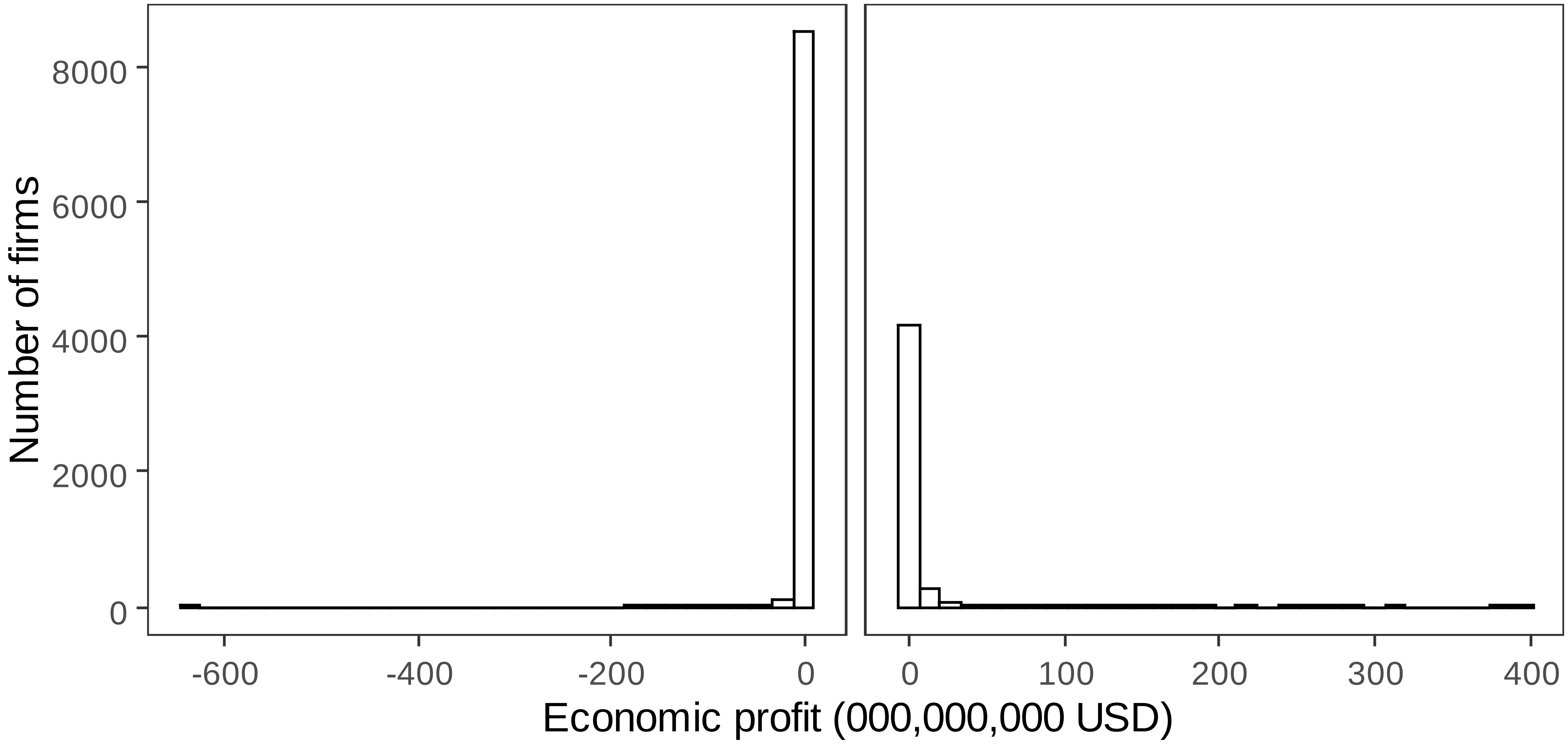
- Sum over discounted economic profit:

$$\sum_{t=1}^{T} \delta_t \, (\pi_t - r_t \, K_t)$$

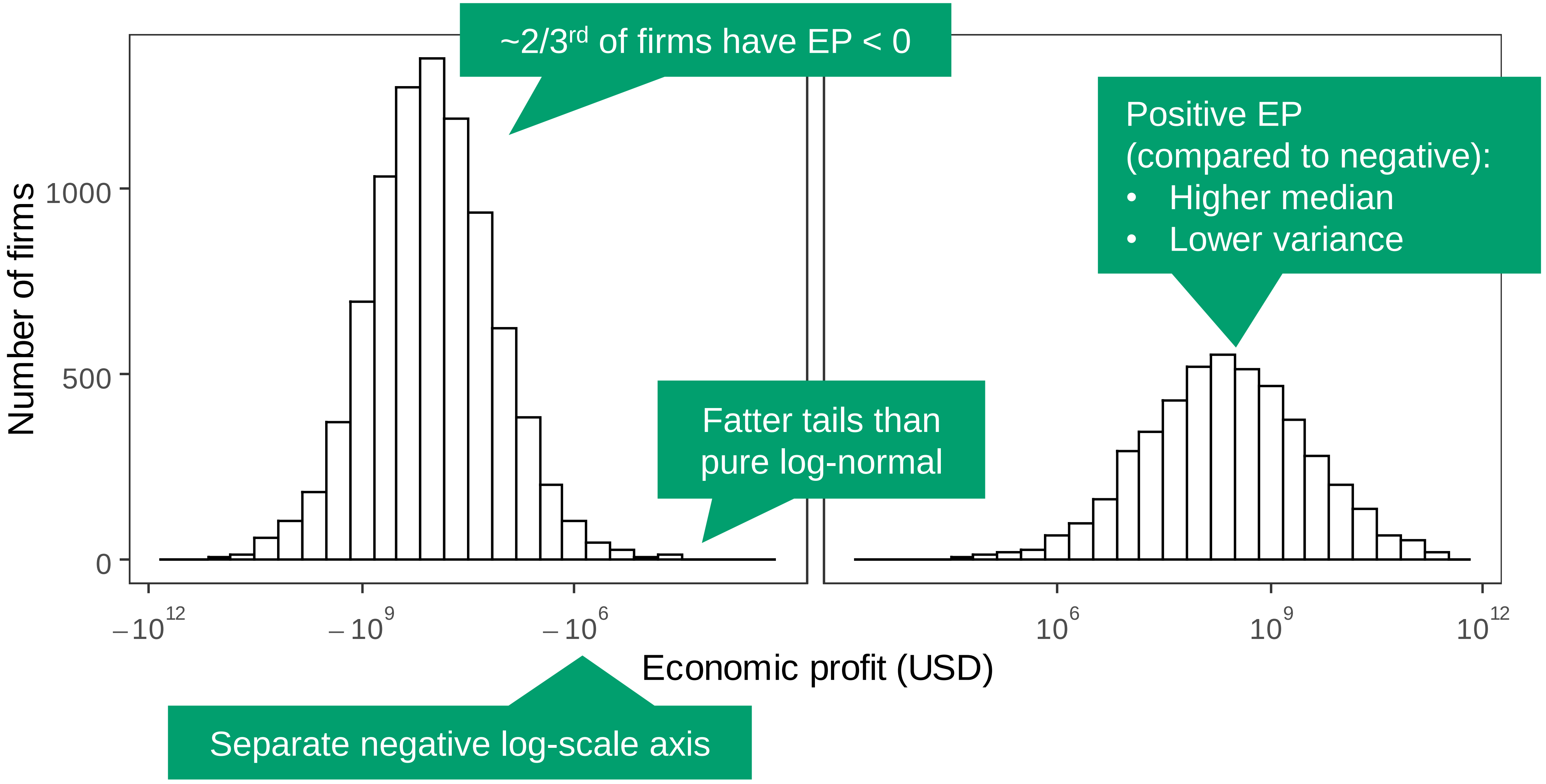
- Variable definitions:
  - π_t = NOPAT (net operating profit after tax) = EBIT – taxes (from Compustat)
  - K_t = capital employed = operating assets – operating liabilities (from Compustat)
  - r_t = cost of capital = annual risk-free rate + risk premium
  - δ_t = discount factor (follows from cost of capital over multiple years)

- Measure is based on LIVA (long-term investor value appropriation) Wibbens & Siggelkow (SMJ 2020)

The distribution of long-term profit (Listed US firms 1999-2018)



The distribution of long-term profit (Listed US firms 1999-2018)



A simple stochastic process fits the data well

- X_t a geometric random walk (as before):

$$\mathrm{d}X_t = X_t(\mu\,\mathrm{d}t + \sigma\,\mathrm{d}z_t)$$

- Let X_0 log-normally distributed

- Distribution of $Y_T = X_T - X_0$ fits the data well

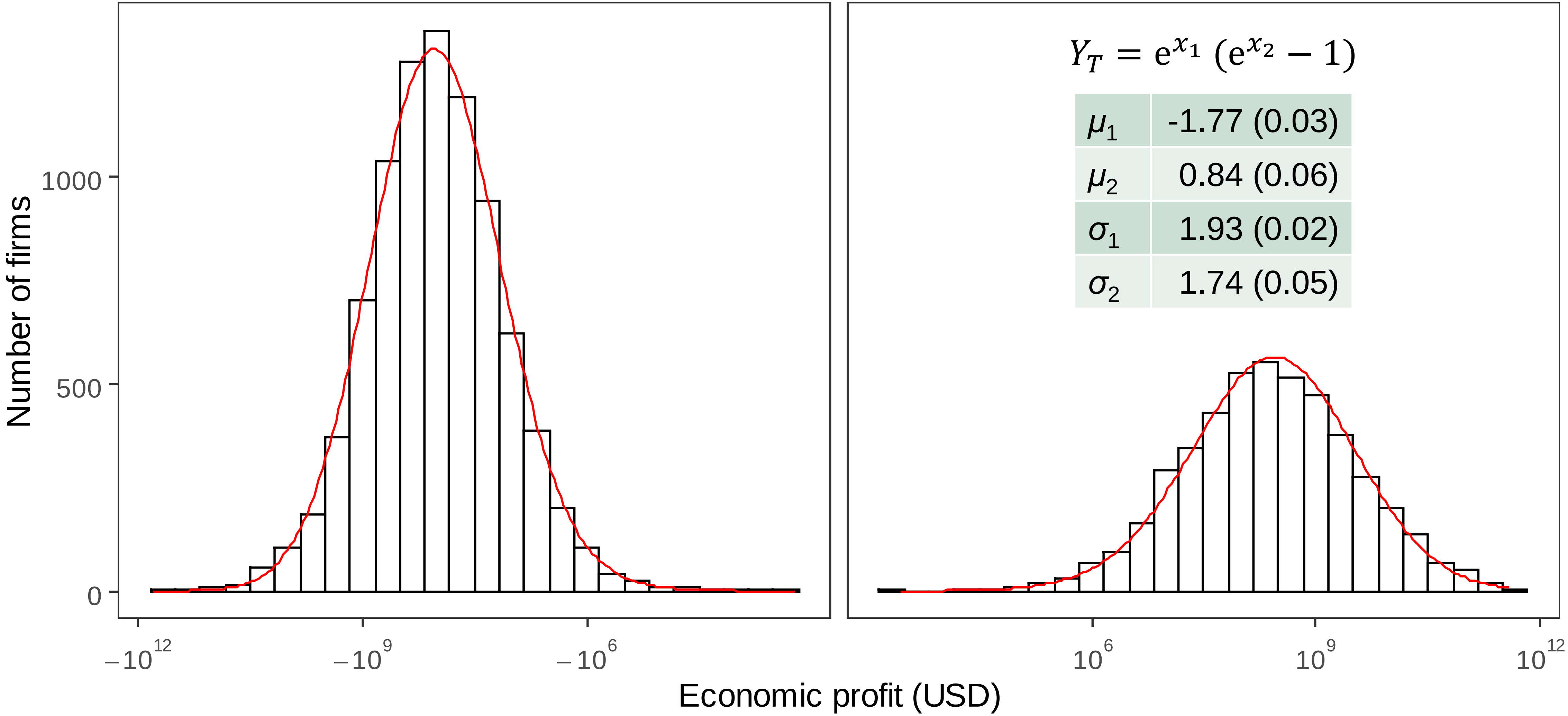
- Alternatively:

$$Y_T = \mathrm{e}^{x_1}\,(\mathrm{e}^{x_2} - 1)$$

$$x_i \sim N(\mu_i, \sigma_i)$$

- Four parameters to be estimated with ML: μ_1, μ_2, σ_1, σ_2

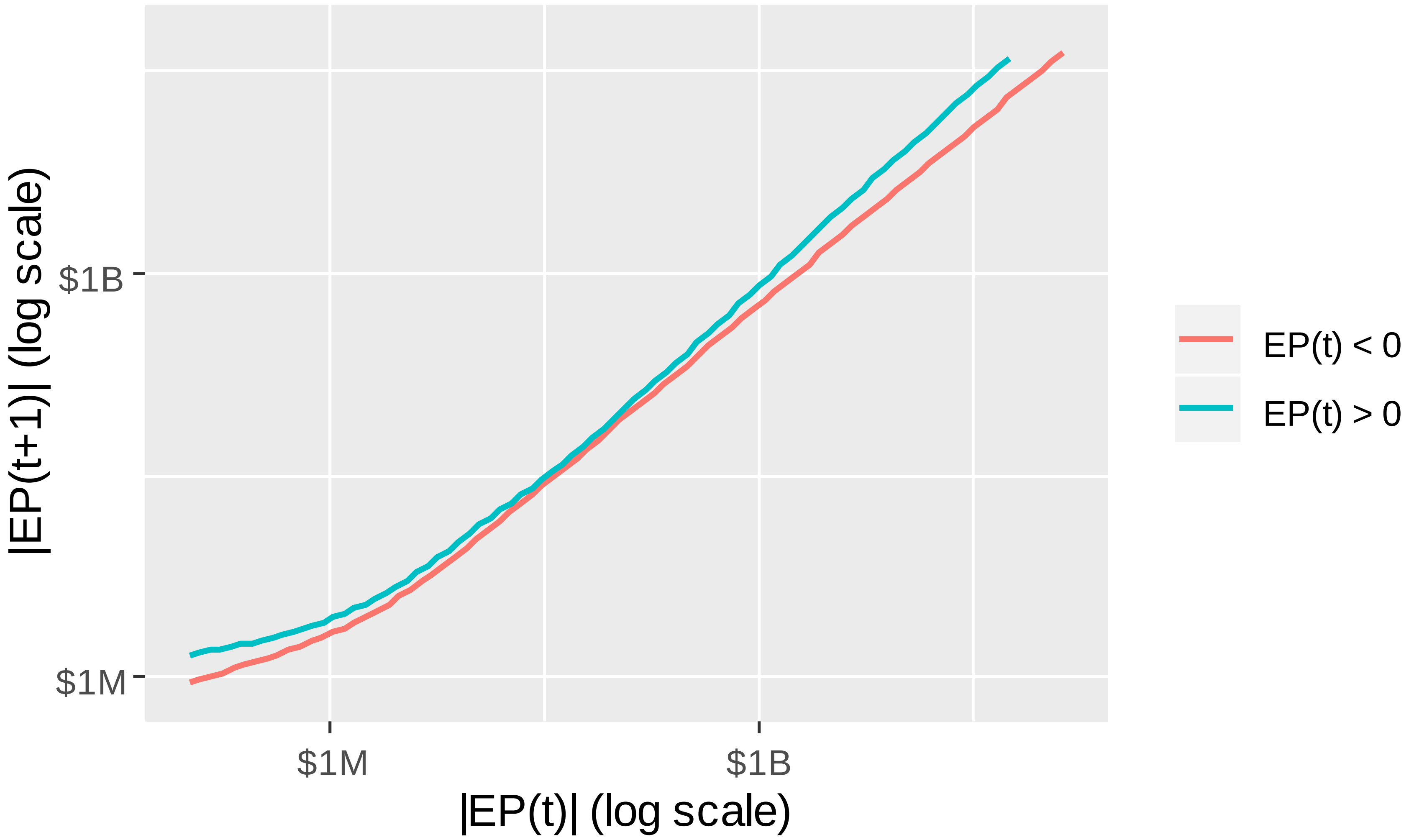
Red line = MLE fit with Y_T distribution



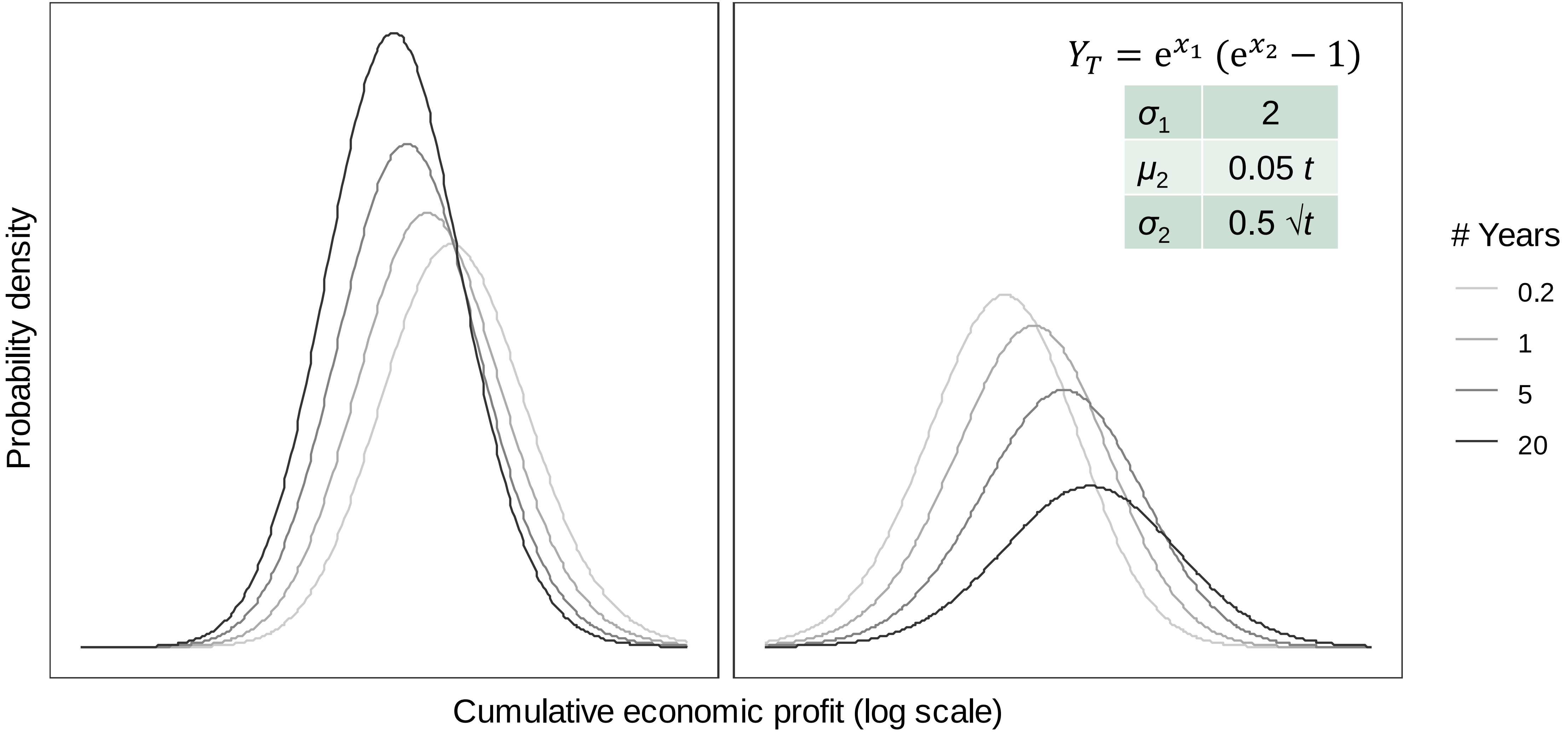
$$Y_T = e^{x_1}\,(e^{x_2} - 1)$$

μ_1	-1.77 (0.03)
μ_2	0.84 (0.06)
σ_1	1.93 (0.02)
σ_2	1.74 (0.05)

If you're unprofitable, it's harder to catch up



Asymmetry of Y_T increases over time



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