Title: The distribution of long-term firm performance

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Abstract: The distribution of firm size and many other economic variables has been well-documented. This study documents the distribution of firm profit, a key variable whose distribution has received little attention despite its economic salience. Across large samples for contemporary US and European firms as well as historic US firms, only between 27% and 37% of firms have been to earn profits above their cost of capital over a twenty-year period. A three-parameter stochastic process based on a geometric random walk describes the distribution of observed profits remarkably well and explains why most firms do not earn their cost of capital. Thus, this study provides a deeper understanding into the generation process of long-term profit, an economic variable that is central to market-based economies.

One Sentence Summary: A simple stochastic process explains why most firms do not earn profits above the cost of capital

Main Text:

Many economic variables, such as firm size, stock market returns and individual wealth, exhibit remarkable regularities in aggregate. Their distributions are often log-normal or follow a power-law (*1-3*). Stochastic processes, such as Gibrat's law, can yield deeper insights into the underlying drivers of such observed empirical regularities (*4*, *5*)

However, a key economic variable whose distribution has received little attention is firm profit. This omission is particularly striking since generating profits is a key goal for firms and thus a critical factor in the decision making of managers and investors. Profits shape market-based economic activity. Therefore, in this study I document the distribution of corporate profits in various samples and present a stochastic model to explain the observed distribution.

There are many ways to define profit, but for economic analysis a version of economic profit—rather than accounting profit—is the most useful, because it reflects whether capital is put to best use (*6*). Here, economic profit is defined as the net operating profit after tax (NOPAT) minus the cost of capital times the balance sheet invested capital (*7*). Because investors are primarily interested in optimizing long-term value rather than single-year profits, I aggregate profits over several years. In an equation, the economic profit for a firm i over years t is defined by:

$$\mathrm{EP}_i = \sum_t \delta_t \left[\mathrm{NOPAT}_t - (\text{cost of capital})_t \times (\text{invested capital})_{it}\right]$$

In this equation, δ_t is a factor that takes into account that profits generated earlier are more valuable, because they could be reinvested to generate even more profit in later years (*8, 9*). All distributions pertain to this calculation of long-term economic profit. More details are available in the Supplemental materials (*10*).

The bars in Fig. 1 represent the observed distribution of economic profit of all listed firms in the United States over the past 20 years, based on the reported EBIT (earnings before interest and taxes) and balance sheet capital in the Compustat database. Note that, unlike for instance company size, profit can also be negative and therefore can never be described with a single log-normal or power-law distribution. Because profits do vary over many orders of magnitude, it is still useful to display them using logarithmic transformations, separately for the positive and for the negative axis.

It might seem that the observations in Fig. 1 can be best described using two separate log-normal distributions for positive and for negative profits. However, log-normal distributions do not accurately describe the behavior at the tails close to zero. Moreover, using two separate distributions is not a very satisfying solution, because it does not account for various peculiar characteristics of the distribution:

1. Most firms (73%) have a negative economic profit
2. The median of positive profits is more extreme than that of negative profits ($223M vs. -$166M)
3. The variance of positive profits is higher than that of negative profits

Are these three characteristics merely an oddity of this distribution, or do they result from a deeper process? The latter, as it turns out. A simple stochastic process can explain these stylized facts without assuming them and matches the observed data remarkably well. The process is, like Gibrat's law, based on a geometric Brownian motion $\mathrm{d}X_t = \sigma X_t \,\mathrm{d}z_t$, which states that the change in a quantity $\mathrm{d}X_t$ is proportional to its size X_t.

Here, the variable X_t can be interpreted as accumulated value, while the difference $Y = X_T - X_0$ between the value at two time points $t = 0$ and $t = T$ represents the economic profit (*11*). Assume moreover that X_0 is distributed log-normally with parameters μ_0 and σ_0, consistent with the fact that a geometric Brownian motion leads to log-normal distributions in the cross-section. The derivation in the Materials and methods of this study's Supplemental materials (*10*) shows that the distribution of the resulting economic profit can be described in terms of two independent draws of normally distributed variables:

$$Y = \mathrm{e}^{x0} (\mathrm{e}^{x1} - 1)$$

$$x_0 \sim N(\mu_0, \sigma_0)$$

$$x_1 \sim N(-\tfrac{1}{2} \sigma^2 T, \sigma \sqrt{T})$$

The red line in Fig. 1 shows the density function of Y as defined above, using $T = 20$ years and the maximum likelihood estimates for the parameters μ_0, σ_0, and σ. It matches the empirical distribution well. A formal model selection test confirms that the model based on the above stochastic process better explains the data than a model based on two log-normal distributions (*12*).

To illustrate that this distribution is not particular to contemporary US profits, Fig. 2 shows the empirical and implied distributions for three other data sets: Panel A for historic corporate profits, over the period 1970-1989; and Panel B for profits of firms in the Euro zone. The model fits quite well for these three data sets too, all exhibiting the earlier mentioned three characteristics for the distribution of negative vs. positive economic profit.

The close alignment of the above stochastic process with the observed data is all the more remarkable since firm growth rates exhibit deviations from the geometric Brownian motion as described in Gibrat's law (*1, 13, 14*) and moreover accounting profits, unlike firm growth rates, exhibit a clear persistence over time (*15, 16*), violating the proportional growth assumption of Gibrat's law. Apparently, these deviations do not affect the distribution of profits in the long run. This is likely driven by two factors. First, accounting returns in profit persistence studies are measured relative to balance sheet assets and can provide a distorted measure of underlying economic performance (*17*), while the process here is based on the 'true' accumulated value X_t, which in general is empirically unobservable. Second, deviations from the proportional growth assumption might cancel out in the long run, both through the method of aggregating economic profit over a long time period (*9*) and through statistical aggregation, which tends to lead to normally distributed variables as implied by the central limit theorem (*18*).

Moreover, the model provides deeper insight into the processes shaping long-term profit, in particular why there are so many more firms with a negative than with a positive economic profit. Consider a firm that faces a negative shock to its profits $\Delta X_t < 0$. That shock will not only reduce its profits Y, but also its size X_t and thus the size of future shocks to its profitability, which are proportional to its size. Hence, after a negative shock it will become harder to catch up. Conversely, a firm that has enjoyed a positive shock to its profit, will in a future period receive a larger shock size, and thus have a higher probability to generate extremely positive profits. Thus, the evolutionary process in which more profitable firms on average grow faster shapes the distribution in which most firms earn negative returns and few firms generate positive profits but with more extreme values.

Finally, the parameter estimates of the model provide further insight in the profit generation process in various settings. Fig. 3 shows the maximum likelihood estimates with 95% confidence intervals of the three model parameters for each of the four data sets presented in Fig. 1 and Fig. 2. Though differences in μ_0 merely represent size differences, the differences in σ and σ_0 represent fundamental differences in the profit generation process.

Most notably, the annualized standard deviation σ of the geometric Brownian motion for US profits over the most recent two decades is significantly higher than for both the US profits in the 1970s and 1980s and the contemporary profits in the Euro zone. These differences in σ are directly related for the fact that a much smaller share of current US firms is able to generate positive economic profits than historically or in the Euro zone: 27% in the US currently, compared to 37% in the US 1970-1989, and 38% in the Euro zone. Apparently, in the current US economic environment the difference between winning and losing firms is more pronounced than historically or in Europe, potentially due to the strong network effects in IT industries (*19*).

More generally, this study reinforces the power of using stochastic processes to understand aggregate results. With some modifications, the geometric Brownian motion can not only explain the distribution of firm size, but also the—arguably even more important—distribution of firm profit.

References and Notes:

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10. Materials and methods are available as supplementary materials at the Science website.
11. Note that the stochastic process has zero drift, and hence the expected value of economic profit Y has zero expected value. This is true in general, because economic profit measures returns relative to average. For a more elaborate argument, see P. D. Wibbens, N. Siggelkow, Introducing LIVA to Measure Long-Term Firm Performance. Academy of Management Proceedings 2017, 11325 (2017).
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Acknowledgments: The author thanks Dan Levinthal and Phanish Puranam for helpful conversations and comments on the paper. **Funding:** the author has no funding to declare. **Author contributions:** P.W. performed the conceptual development, analysis, and writing of the paper. **Competing interests:** “Authors declare no competing interests. **Data and materials availability:** Data are based on the widely used Compustat and CRSP databases, used here through WRDS (Wharton research data services). R scripts to reproduce the analysis using these databases are available in the supplementary materials.

Supplementary Materials:

Materials and Methods

References (##-##)

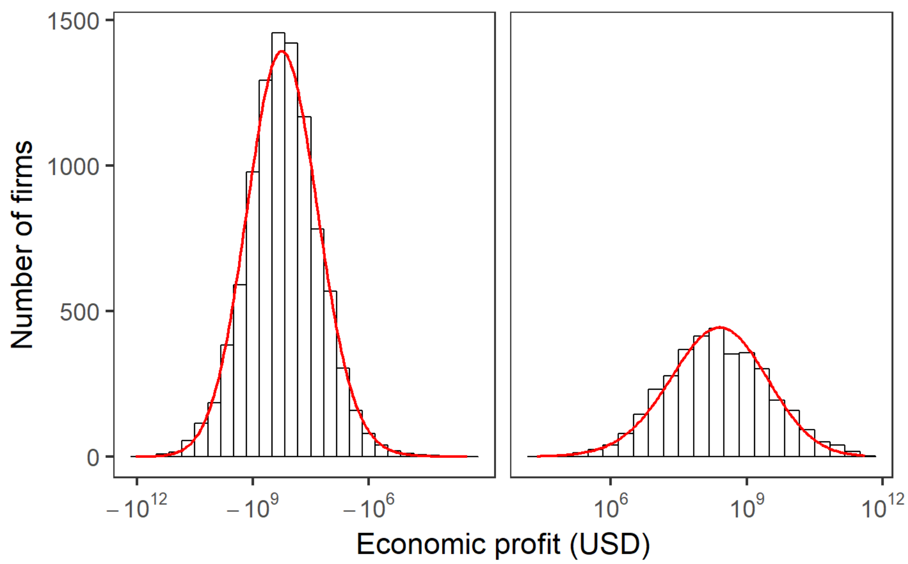

Fig. 1. Distribution of long-term economic profit in the US. Bars represent observed distribution for all US listed firms over the period 1999-2018 (n = 13,282). Red line represents the distribution implied by the stochastic process discussed in the text. The x-axes are on a logarithmic scale.

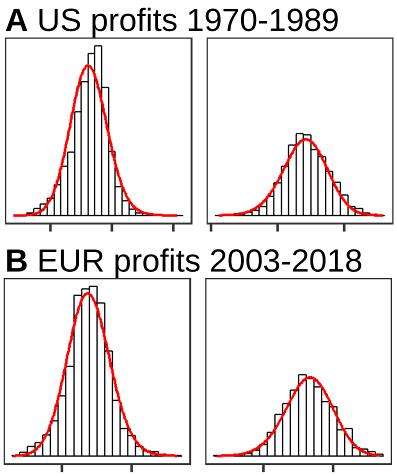

Fig. 2. Observed and implied distributions in other samples. Definitions of bars, lines, and axes as in Fig. 1.

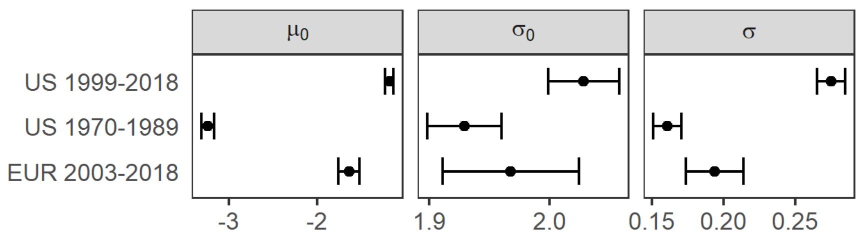

Fig 3. Parameter estimates. Maximum likelihood estimates (dots) with 95% confidence intervals (error bars) for the model as discussed in the text based on three different data sets. μ_0 and σ_0 parametrize the initial log-normal distribution, while σ represents the *per annum* standard deviation of the geometric Brownian motion.