

Intellectual Property Rights, Professional Business Services and Earnings Inequality

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Abstract: Need for high-skilled labor changes over time but firms cannot adjust perfectly due to several adjustment costs of hiring and firing. The Professional Business Services (PBS) sector alleviates this problem by allowing high skill labor to move across firms, pooling idiosyncratic part of labor demand risk. This allows for better utilization of high skill talent and increases their productivity. This paper aims to show that improvements in intellectual property rights protection in the late 1970s alleviated concerns regarding sharing sensitive data with 3rd party firms, thus helped the PBS to thrive in the following decades. Better utilization of high skill increased productivity in the economy. By increasing the compensation of professionals, it also contributed to the increased earnings inequality since 1980. A panel data analysis using time variation in the U.S. states' adoption of the Uniform Trade Secrets Act shows a positive association between the trade secrets protection and the PBS employment, where adoption is associated with 10% higher employment on average. We build a model that is able to match the qualitative characteristics of the evolution of earnings inequality in the U.S. since 1980, using this mechanism.

Key Words: Earnings Inequality, Intellectual Property Rights, Organizational Structure, Structural Transformation, Trade Secrets

JEL codes: J31, K22

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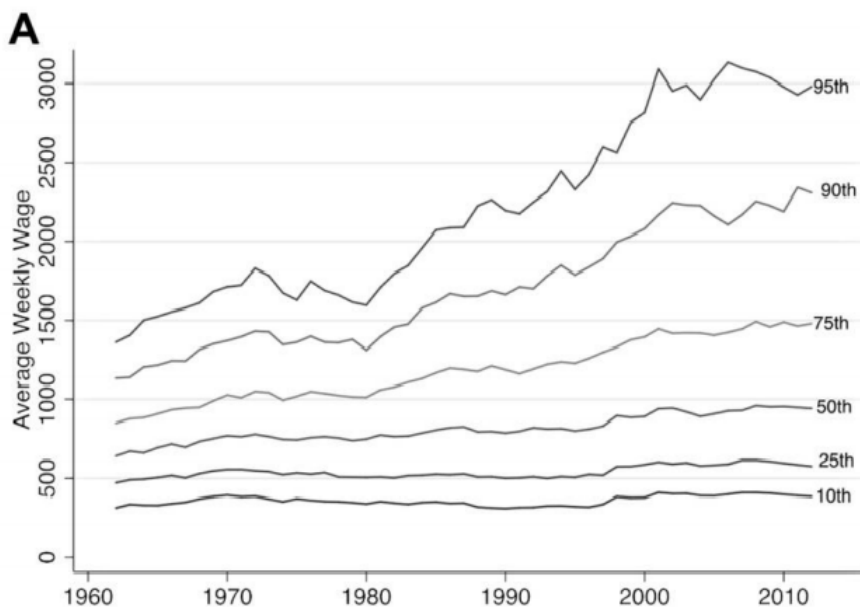
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***** PRELIMINARY AND INCOMPLETE DRAFT *****

1 Introduction

Increasing inequality in the U.S. has become one of the major issues in the past decades. There is a large body of research on both the causes and effects of increased inequality, which recently gained pace with the availability of better data. Still, there is no consensus on the economic forces behind and the magnitude of their contribution.

In this paper, we are interested in increasing earnings inequality. As seen in Figure 1 taken from Murphy and Topel (2016), average weekly wage of full-time employed males has been more or less steady across income quantiles until 1980. While the inequality below the median continued to remain steady, inequality above the median has risen considerably during the ‘80s and ‘90s. This graph alone generates many interesting questions, answers of which are central for policies aimed at reducing inequality. Is it the distribution of talent that is changing or is it how different levels of talent are compensated? Why does the divergence only happens at the top half? What caused the structural break around 1980? A theory of earnings inequality has to account for the stable regime until 1980 for the whole distribution with a steady increase in inequality at the top half after 1980.



We contribute to solving this puzzle by proposing a novel mechanism: increased utilization of high skilled labor through Professional Business Services (PBS) sector. This sector consists of firms who provide services to other firms, employing high-skill individuals¹. The

¹Consulting firms, law firms, advertising firms, and information technology (IT) firms would be some of the examples.

mechanism builds on two premises. First, a large part of the tasks that high-skill individuals perform is infrequently needed by firms. Second, to utilize high-skill individuals, firms need to share sensitive information (costs, customer lists, product designs etc.) with them. Absent any frictions, firms would find it optimal to hire these experts outside their daily business activities only for the period they actually need them and finalize their employment afterward. However, there are large hiring and firing costs involved with this practice, which makes this infeasible in most situations. Thus, firms either would hire these experts long term, taking into account that their expertise will not be useful when there is no need for them or would not hire them at all.

As long as there is some idiosyncrasy in different firms' demand for a certain expertise, a natural solution to this problem would be an intermediate firm that hires the experts and rents their services for a short time to demanding firms². However, firms have to provide rented experts with sensitive information. Firms will be hesitant to do so if they are not certain that these intermediate firms are willing and able to protect this sensitive information from competitors. If the courts are not well-equipped to enforce this protection, intermediate firms could only rely on reputation, which makes it very hard for new firms to enter and the sector to grow.

This paper aims to construct two important causal links and measure their quantitative importance. First, the improvements in intellectual property (IP) rights around 1980 has contributed to the growth of the PBS sector. Second, the growth of the PBS sector has improved the allocation of high skill talent across firms, thus increased their compensation. Consequently, earnings inequality has grown.

Coinciding with the start of increasing wage inequality, the late '70s and early '80s was a period in the U.S. with major improvements in the IP law. Economists have specifically studied the improvements in patent protection³. For our purposes, we focus on a less studied but potentially more important improvement: the signing of Uniform Trade Secrets Act (UTSA) in 1979. While patents protect firms' ability to enforce patented innovations, only a small fraction of intangible capital in the U.S. is patented and a small fraction of companies hold any patents at all. On the other hand, there is a mountain of secret information just as important for keeping a competitive edge: costs, pricing strategy, production technologies, client information, long-term strategies etc. Even explicit R&D projects are trade secrets until they are successfully finalized and patented. Moreover, many firms choose to keep their

²An individual contractor supplying her services to different firms for short periods would also serve the same purpose.

³See Henry and Turner (2006), Han (2018), etc.

innovations secret even if they can be easily patented, thinking these innovations worth more to them when they are kept secret⁴.

Before UTSA was enacted, there was no statutory law for the protection of trade secrets. Courts were relying on torts, previous court rulings and competition law to settle disagreements. Consequently, every state's court system was following a different procedure. Enforcement of trade secret protection was unpredictable, and not uniform across the U.S. courts. UTSA provided a set of rules that states could follow in trade secret protection. States adopted the UTSA at different times ranging from 1980 to today. UTSA not only improved trade secret protection in the states it was adopted but also in other states by leading to precedent court decisions in adopting states that other states could follow.

End of the '70s was also a period where PBS started to enjoy an unprecedented rate of growth, outpacing all large industries through '80s and '90s. While PBS employed 9% of the U.S. labor force in 1980, it employed 14% in 2017. This is important for the allocation of resources in the economy since PBS firms act as a buffer against labor adjustment costs of other firms. They supply high skill labor to firms when they need it without a long-term commitment. Therefore, a high skill employee whose services are infrequently required can work for multiple companies and be productive throughout her employment. Consequently, a professional of similar human capital level can earn more in an economy where the PBS sector is more developed. Moreover, this also allows high skilled individuals who are employed in this sector to accumulate human capital at a faster rate, both by working more and by working and studying a multitude of different problems arising in multiple firms. This makes them particularly attractive for managerial positions, where firms value broad business experience.

Lower skilled business services are not affected by the proposed mechanisms due to two reasons. First, the need for most of these services (security, cleaning, cooking etc.) does not change over time in the usual business operation of firms. Therefore, the gain from being able to relocate low-skill labor is not as large. Second, they usually do not need sensitive information to be able to perform their duties, therefore trade secret protections do not have direct implications for them.

Our regression analysis shows a positive and significant association between PBS employment and quality of trade secret protection, while no such association exists for non-professional business services. The average increase in trade secret protection index only

⁴Famous examples are the formula of Coca Cola and DuPont's Kevlar technology. See Horstmann et al. (1985) for a model of patenting vs not patenting decision of firms.

due to the adoption of UTSA is 0.43, which corresponds to a 10% growth in employment according to our estimates. We build a model that incorporates the mechanism and is able to match important characteristics of the evolution of earnings inequality in the U.S. starting from 1980. Quantitative analysis of the model is to come.

1.1 Literature Review

This paper contributes to three pieces of literature: increasing wage inequality, drivers of structural transformation and drivers of declining business dynamism. For the first literature, we provide a novel channel that contributes to higher wage inequality. Explanations proposed in the literature for these phenomena can be summarized in two categories: changes in skill distribution and changes in how skills are compensated. In the first category, researchers find some of the increasing wage inequality can be explained by compositional changes. The second category can be further divided into two sub-categories, namely, changes in how skills are utilized in the economy and changes in how the surplus from production is shared across skill levels. In the first sub-category, the main proposed explanations are changes in the relative demand for different skill groups through technological advancements, increased outsourcing/off-shoring of low-paying jobs and changing managerial hierarchies in firms. In the second sub-category, main proposed explanation is decline in unionization⁵. This paper fits into the literature that analyzes the changes in how agents with different skill levels are utilized in the economy. Increased utilization of high skilled labor by improved allocation across firms through renting technology has not been provided as an explanation for the relevant labor market trends, to the best of our knowledge.

The structural transformation literature tries to explain the major growth of the services sector in the U.S. economy after 1950. The first branch tries to explain the rise in services by shifts in preferences towards market goods versus home production with increasing income levels by assuming non-homothetic preferences. Some of the papers here also have implications for wage distribution since these trends bring increasing specialization in the services sector. Second branch shows how differential technological growth rates across industries could have brought structural changes⁶. The papers in the first branch mainly focus on

⁵As examples, see Lemieux (2006) for composition of the labor force, Acemoglu and Autor (2011) for skill-biased technological change, Elsby et al. (2013) for offshoring, Goldschmidt and Schmieder (2017) for outsourcing, Garicano and Rossi-Hansberg (2006) for organizational hierarchies and Farber et al. (2018) for unionization.

⁶See Kongsamut et al. (2001) and Buera and Kaboski (2012) for the income effect and Lee and Wolpin (2006), Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008) for the technological effect.

consumer services. As shown in the next section, the growth in PBS was much larger than the rest of the services, which cannot be explained by consumer preferences or an increase in overall productivity which affects all the economy in the same way. This paper contributes to the second branch by introducing a novel channel which would contribute to differential productivity growth rates across sectors: easier business partnership through improvements in the protection of intellectual property. Herrendorf et al. (2013) shows, differential growth rates have contributed to growth in the ‘Value Added’ share of services more than the ‘Final Consumption’ share. This is in line with our suggested mechanism, as the growth of the PBS would show up in the trend of ‘Value Added’ share of services but would not have a direct effect on the trend in ‘Final Consumption’ share.

The literature on declining business dynamism analyses two important trends in the U.S. economy: the decline in entrepreneurship and job-to-job transitions⁷. Both trends are perceived as concerning. More entrepreneurial activity is usually associated with more new ideas, more creative destruction, and higher growth. High job-to-job transition rate is usually associated with a better allocation of resources across firms, therefore higher productivity. Our mechanism would imply both of these trends as a result of the growth of the PBS. First, increased compensation of high-skill individuals in the PBS sector increases the compensation of all high-skill employees by providing a better paid outside option. This, in turn, increases the opportunity cost of an entrepreneurial activity, reducing the incentives of high-skill for opening up a new firm. Therefore, growth in the PBS sector might impede new ideas from coming into the marketplace. Second, being able to switch workplaces without the need for changing employers reduces the need for job-to-job transitions. Idiosyncratic demand for high-skill labor can be met by renting individuals instead of hiring them with the existence of a PBS sector. Thus, the decline in job-to-job transitions might not be as worrisome as previously thought and could be a result of a more flexible employment agreement.

The results of this mechanism are also in line with a new set of facts that have been put forward in the last decade using newer and better data sources that became available to economists. First, Barth et al. (2016) uses matched employer-employee data from the U.S. Census Bureau and shows that the increase in earnings inequality mainly occurred between firms rather than within firms. Song et al. (2018) uses income tax reports for the U.S. and reaches the same result, while Card et al. (2013) finds a similar result for West Germany using matched employee-employer data. Song et al. (2018) further shows the increase in between-

⁷See Decker et al. (2014) for the decline in entrepreneurial activity and Molloy et al. (2016) for the decline in job-to-job transitions.

firm inequality is almost completely due to increased sorting. As PBS firms are formed, they become hubs of high-skilled high-earning employees, becoming ‘superstar firms’ as denoted in Autor et al. (2017). This would naturally increase the wage inequality between firms while having smaller effects on within-firm wage inequality. Second, Professional business firms tend to be formed as partnerships or as S-corps, instead of C-corps to utilize a more lenient tax structure. This structure makes it more profitable to make a productive employee a partner, reducing the effective tax rate on company earnings. This supports the results of Smith et al. (2017), which uses income tax records to conclude most of the increase in income inequality was coming from a growing private business income of active managers. Partners of PBS firms perfectly fit this description.

The rest of the paper is structured as follows. Section 2 introduces the background information on trade secret protection and PBS sector and discusses the data availability. Section 3 has the empirical analysis that forms the connection between improvements in trade secret protection and PBS sector growth. Section 4 presents the static model and Section 5 introduces the dynamic model. Section 6 discusses the estimation strategy while Section 7 discusses interesting extensions to the model and Section 8 concludes.

2 Background and Data

2.1 Trade Secrets and Uniform Trade Secrets Act

Trade secrets are all the information businesses have that grants them a competitive advantage over their rivals when they are kept secret. Information on business relations and practices such as customer lists, pricing strategy, cost information, and long-term strategy as well as R&D related information such as formulas, manufacturing techniques and designs can be trade secrets. The former type of secrets is valuable only when they are kept secret, while the latter type can still generate value through patenting. However, (1) not all innovations satisfy conditions of patentability ⁸, (2) firms choose not to patent some of their innovations thinking it is more profitable to keep them secret and (3) any innovation process has to be kept as a secret until it is developed enough to be able to be patentable. While patenting requires lawyers, time and financial resources even before any protection is there, trade secrets only require taking precautions to protect secrecy. Thus, trade secrets are especially important for small to middle sized firms.

⁸The innovation has to be considered “useful,” “novel,” and “nonobvious” by the patent office.

Trade secrets are seriously understudied compared to other forms of intellectual property such as patents and trademarks, as measuring the value of secrets is hard. While a small fraction of U.S. businesses holds patents, all firms carry some type of trade secrets. Surveys by Cohen et al. (2003), Arundel (2001) and Marsh and Liberty Underwriters (2011) show firms on average value trade secrets the highest among their intellectual property (IP), before patents or trademarks. Trade secrets are also the most litigated form of intellectual property (Lerner, 2006). While other IP law cases are governed by federal statutory law, trade secret laws vary state by state. In addition, all patent appeals go to U.S. Court of Appeals for the Federal Circuit, making the statistical analysis of patent cases much easier. Trade secret cases, on the other hand, can be heard both in state courts and federal courts, therefore there is no single entity that researchers can focus on. Furthermore, state courts rarely publish explanations for given decisions. All these problems limited statistical research on trade secret protection.

It has been documented in several other studies that trade secrets are very important for firms. We aim to convince the reader on two other points here: (1) UTSA had a significant impact on trade secret protection and (2) trade secret protection is crucial for PBS dealings. In subsection 2.1.1 we provide evidence for the former and in subsection 2.1.2 for the latter.

2.1.1 Uniform Trade Secrets Act

Before 1979, protection of trade secrets was established through common law⁹. This created two main problems. Firstly, as no two cases are the same, there was a large uncertainty regarding what could be designated as a trade secret, what would constitute misappropriation and the size of the remedies in case a misappropriation has been found¹⁰. Secondly, trade secret protection varied substantially across U.S. states, creating further uncertainty whenever sides were not established in the same state. This problem has become even larger when the Supreme Court has ruled that state courts cannot use decisions made by federal courts as common law in *Erie Railroad Co. v. Tompkins* (1938) case. This landmark decision meant each state could only rely on the decisions made by their own courts, removing the only possible unifying body from the picture¹¹.

⁹Common law, as opposed to statutory law, does not rely on an explicit set of rules set by lawmakers. Instead, it uses decisions made in previous court cases to reach new decisions.

¹⁰"... even in states in which there has been significant litigation, there is undue uncertainty concerning the parameters of trade secret protection, and the appropriate remedies for misappropriation of a trade secret.", UTSA Prefatory Note (1985).

¹¹Edward S. Rogers, who was chairman of the board of executives of Sterling Drug Co. and a member of Lawyers' Advisory Committee of U.S. Trademark Association would later say "Nobody knew what the law

In order to deal with these problems, the American Law Institute has published several 'Restatements of Torts', which were a small number of rules that summarized the general theme of previous decisions. However, the statements were necessarily vague where uncertainty was the highest and had no legal binding on how courts were going to decide. Furthermore, there were two important missing points that would make Restatement of Torts unfit for dealings between PBS firms and their clients. First, it required the data to be illegally appropriated. Second, the accused party had to be in direct competition with the plaintiff. Neither of these requirements would be satisfied in general between PBS firms and their clients.

After a lengthy lawmaking process, UTSA has been proposed in 1979. Each state has to opt-in to use UTSA for it to be effective in state courts. Minnesota, Idaho, Arkansas, Kansas, and Louisiana were the first states to adopt it in 1980, where UTSA became effective in 1981. By 1988, 26 states had already adopted it. By 2012, only three states had not adopted it. Texas did in 2013, New York and Massachusetts currently are evaluating bills regarding adoption of UTSA, after which all the U.S. states will have adopted UTSA.

Almeling et al. (2010) analyzes 394 cases in which federal district court issued a written opinion between 1950 and 2008 in and Almeling et al. (2011) analyzes 358 state appellate court decisions (issued by intermediate courts of appeal and the highest courts of each state). Although neither group is a random sample of the universe of decisions made regarding trade secret violations, both carry significant detail regarding the decision process of the courts.

The effect of UTSA can also be seen clearly in two series they analyze: the number of decisions per year in Figure 1 and share of cases that use statutory law instead of common law in Figure 2¹².

Figure 1 implies that the number of court cases has seen a major increase starting with the introduction of UTSA. There can be two explanations, both of which support our theory. It might be the case that trade secret violations ended up in courts more often than before or it might be that the number of trade secret violations increased, possibly due to increased sharing of sensitive data. Both explanations would imply that the trade secret protection given by the courts has improved. Figure 2 depicts what percent of the cases seen has used the statutory law instead of the common law. Before 1980, this share is naturally 0%. What this graph shows is that after the introduction of UTSA, the majority of cases was finalized

was. It was frequently found that there were no applicable State decisions or that the decisions in the States comprising the same circuit were not uniform.”.

¹²Both figures are taken from Almeling et al. (2010)

Figure 1: Number of Decisions Made by Federal Courts Regarding Trade Secret Violations 1950-2007

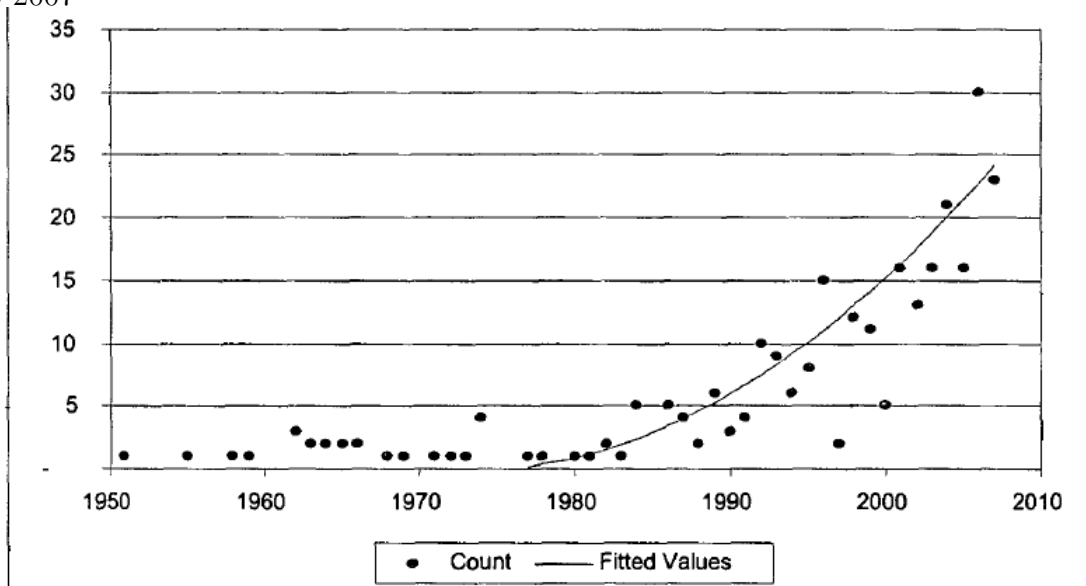
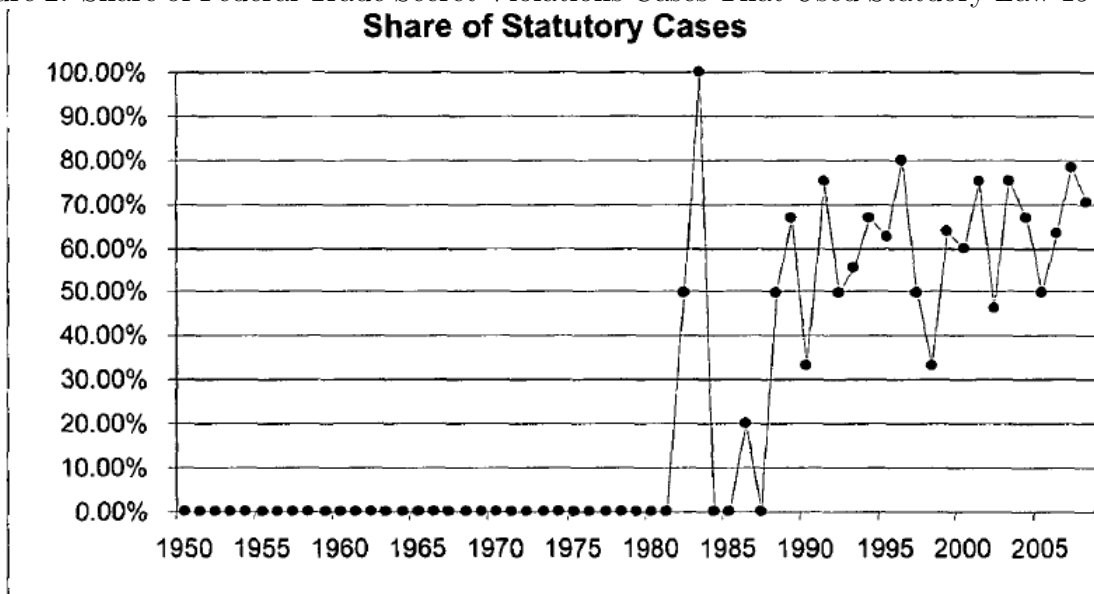


Figure 2: Share of Federal Trade Secret Violations Cases That Used Statutory Law 1950-2007



using the framework introduced by UTSA¹³. Together, these two figures suggest after the UTSA, there have been significant changes in the number of trade secret cases and how these cases were interpreted in courts.

2.1.2 Trade Secret Protection and Professional Business Services

Since IP protection and trade secrets are often mistakenly assumed to be only relevant for secret R&D projects, PBS sector is not associated with informational frictions in the literature. However, previous research shows (1) a large portion of trade secret theft happens with data that would be handled by PBS, (2) a large portion of trade secret theft is done by business partners, (3) dealing with these issues requires a non-trivial legal effort from the defendant firm and (4) cases of trade secret violation has grown with the growing PBS sector.

In 2001, ASIS International has conducted a survey on propriety information loss in the U.S.¹⁴. On-site contractors tend to be reported as the third most important risk factor behind former employees and foreign competitors. On average, 43% of the dollar losses come through information loss in R&D and 38% in financial data. Information loss in customer lists, strategic plans, financial data constitutes half of the total reported incidents in the survey. On average, firms outside high-tech industries consider increased legal costs as the biggest problem created by propriety information loss.

According to Almeling et al. (2010), for cases seen in 2008 (which is the only year that has the whole sample), over 31% of trade secret cases in federal courts, the alleged misappropriator was a business partner. Although trade secrets are usually exemplified with secret formulas, only 9% of the cases involve formulas and only 35 % involve any technical information or know-how. 31% involve customer lists and 35% involve non-technical business information.

According to Almeling (2012), "Only one article about trade secrets appeared in a major U.S. newspaper in the 1970s, but the number of articles on this topic has since mushroomed: 159 articles in the 1980s, 548 in the 1990s, and 593 in the 2000s. Likewise, in the 1970s, there were twenty-six law review articles about trade secrets; by the 1980s that number had

¹³The share is still not 100%. In some situations, the new case in hand might be very similar to a previous case. In these circumstances, the judge might decide to base the decision on the precedent instead of trying to interpret the UTSA from scratch. It is natural that the main beneficiaries of UTSA are other types of cases which do not have close precedents.

¹⁴The survey was done among CEOs of the Fortune 1000 companies and of 600 mid- and small-sized U.S. Chamber of Commerce members. 138 companies have responded. The response bias is claimed to be downward, i.e. firms tend not to disclose information loss or under-report it.

grown to 320 articles, by the 1990s to 1,105, and by the 2000s to 1,546.”

Although none of the presented results here are consequential, they suggest trade secret protection is important in dealings with business partners and consequently with PBS firms. In Section 3, we take these claims one step further analyzing the data.

2.2 Data

2.2.1 Trade Secret Protection

Time variation in states’ adoption of UTSA provides a natural differences-in-differences framework for identifying the effect of trade secret protection on size/growth of PBS sector as well as wage inequality, job-to-job transitions, and productivity¹⁵.

Png (2017a) goes further and construct a trade secret protection index for each state-year, evaluating whether states had certain types of protections and allocates points for each type of protection¹⁶. Figure 3 depicts the behavior of the index for six states across time where solid line is the contribution of adoption of UTSA while the dashed line is the contribution of common law through precedent cases. We use this index as our primary measure for state courts’ ability to provide trade secret protection.

2.2.2 PBS Sector

We have two empirical questions to answer in the paper. First one is regarding the effect of growth in PBS to increasing earnings inequality. The second one is regarding the effect of improved trade secret protection law on growth in PBS. Both questions require relevant firms to employ high-skill individuals, serve businesses and not to consumers. Furthermore, the second question requires protection of client’s private information to be mainly enforced

¹⁵One issue is adoption is not random; states choose when to adopt the proposed act. Png (2017a) claims this adoption process in California and how it got rejected in New York was unrelated to any issues regarding these variables, citing two works by Pooley (1985) and Hutter (1999).

¹⁶”The index is constructed as a simple average of scores for three items of substantive law (i to iii), one item of civil procedure (iv), and two items of remedies (v to vi): (i) Whether a trade secret must be in continuous business use; (ii) Whether the owner must take reasonable efforts to protect the secret; (iii) Whether mere acquisition of the secret constitutes misappropriation; (iv) The limitation on the time for the owner to take legal action for misappropriation; (v) Whether an injunction is limited to eliminating the advantage from misappropriation; and (vi) The multiple of actual damages available in punitive damages. The index is the sum of the scores for each of the six items divided by six, so it is scaled between 0 and 1. For each item, a higher score represents stronger legal protection of trade secrets based on milestones including both common law (decisions in cases that set legal precedent) and the UTSA taking effect.” (Png, 2017a). Png (2017b) extends this measure further until 2010.

Figure 3: Trade Secret Protection Index from Png (2017a)

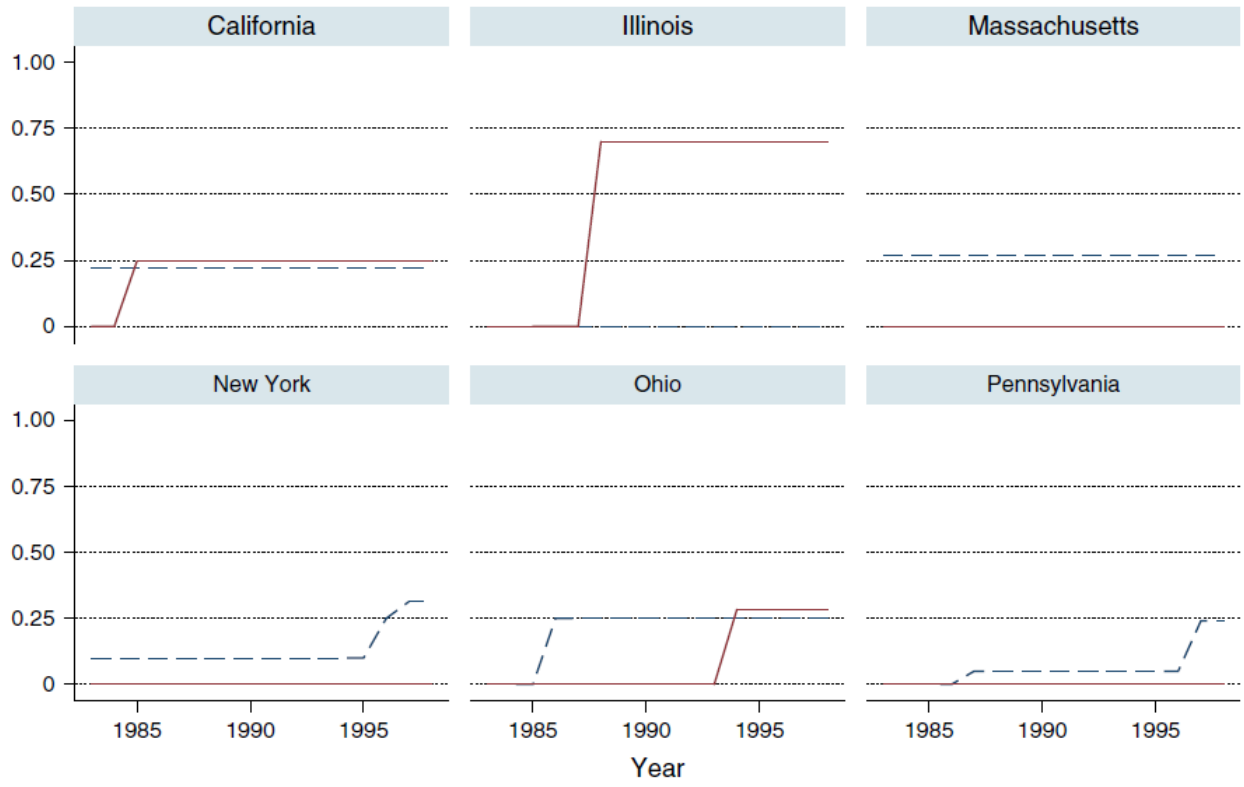


Table 1: Professional Business Services as a subset of 3-digit industry definitions from U.S. Census 1990 classification. Employment data is calculated from CPS.

SIC Code	Subsector	Employment in 2017
721	Advertising	504,601
732	Computer and data processing services	3,372,728
841	Legal Services	1,588,978
882	Engineering, architectural, and surveying services	1,748,358
890	Accounting, auditing, and bookkeeping services	1,179,318
891	Research, development, and testing services	688,093
892	Management and public relations services	1,854,423

through Trade Secrets Law. Thus, the relevant population of firms is not exactly the same for the two questions.

The main dataset we will use for industry employment will be the Current Population Survey (CPS). We chose CPS for having employment data with detailed industry classification going back to the '70s. U.S. Census 1990 3-digit industry classification will be used, which provides decent comparability across time. The 3-digit subsectors that we classify as PBS are given in Table 1, together with the employment levels in 2017. These subsectors can be linked to subsectors under North American Classification System (NAICS) 2-digit sector ‘Professional, Scientific, and Technical Services’¹⁷. Employees in ‘Legal services’ and ‘Accounting, auditing, and bookkeeping services’ are mainly attorneys and accountants, who have occupational client privilege codes. For example, an attorney that discloses her client’s information to 3rd parties would be disbarred by her association. Therefore, protection of trade secrets is mainly carried through occupational codes without the need for an external trade secret law. Consequently, we would expect these two subsectors to be affected to a lesser extent from UTSA, compared to the rest of PBS.

Left-hand side plot in Figure 2.2.2 depicts total non-farm employment, services employment and employment in what we define as the Professional Business Services sector, where employment in each of these is normalized to 100 in 1979. The growth in PBS employment far exceeded the average growth in the US total non-farm and services employment. Moreover, this massive growth started around 1980. Although this graph shows that significance of PBS in US employment structure has increased dramatically and this shift started around the signing of UTSA, there is a long way from here to a causal interpretation. To build the case further, we show how this growth was especially pronounced in those subsectors that

¹⁷See Appendix for a detailed discussion of how these subsectors are selected.

would be most influenced by the signing of UTSA.

Right-hand side plot in Figure 2.2.2 depicts employment in 4 different industry groups, where employment in each of these is normalized to 100 in 1979. The solid one is PBS employment, excluding legal and accounting services. The subsectors included would be the ones that we expect to be most affected by improvements in trade secret protection. The dashed line is the employment in legal and accounting services. The dotted line and the dashed-dotted line has employment in non-professional business services, former calculated exclusively from CPS and latter using both CPS and CES (Current Employment Survey). These trends are in line with our theory, which predicted UTSA to be more influential for those industries where trade secrets have to be shared to a greater extent and trade secret protection is not provided internally.

In summary, following the years after the introduction of UTSA, employment growth in the PBS sector has significantly outpaced comparable groups. Although business services overall have been growing, PBS was particular in its growth rate and timing.

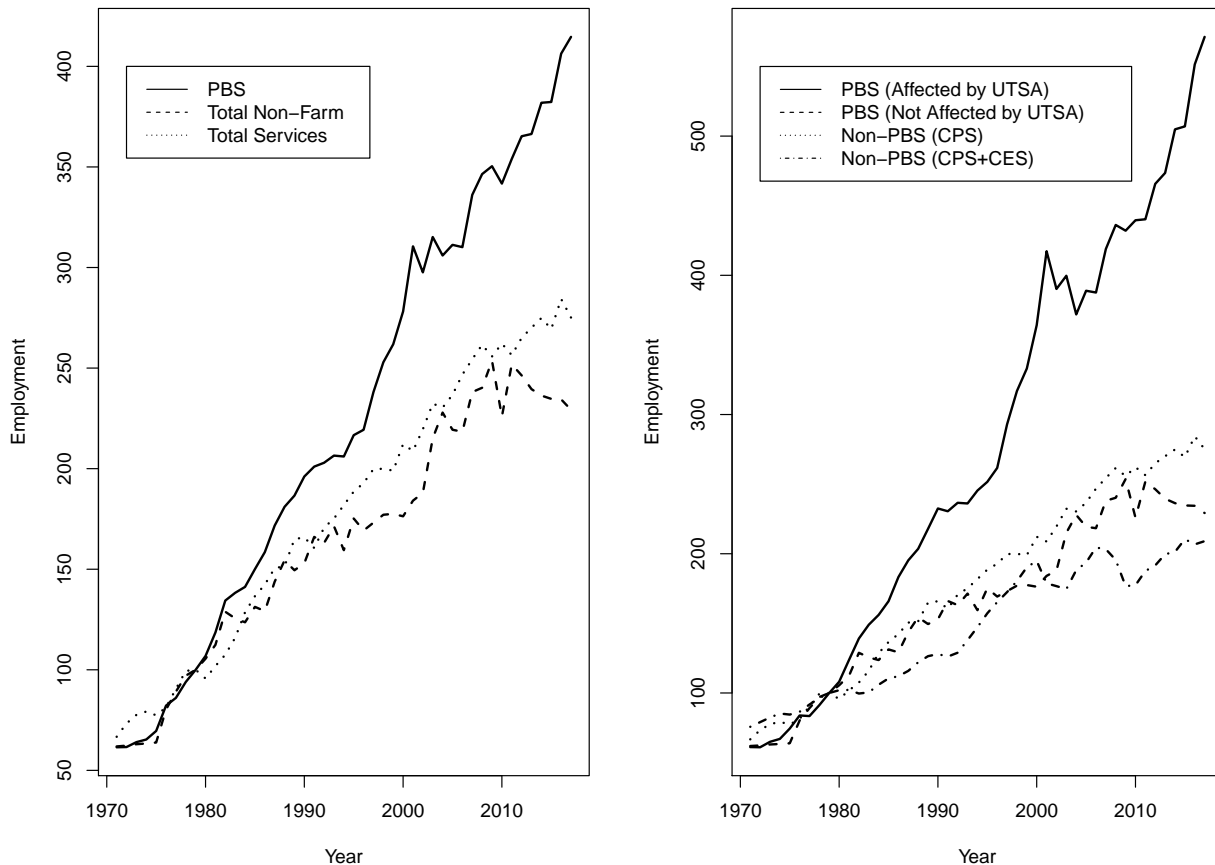
3 Empirical Analysis

In this section, we first run panel regressions to show (1) trade secret protection has a significant positive correlation with PBS employment after conditioning on industry structure in each state-year and (2) it does not have a significant correlation with employment in non-professional business services or legal and accounting services. Second, we use the panel structure to argue it is the law change that drives the growth in the PBS employment and not the other way around. Both empirical results are in line with our theory: for those business service sectors where classified information has to be shared and protection is provided through the legal system, increased trade secret protection is associated with higher employment in the sector.

Our dataset consists of output, employment, and trade secret protection data for 51 states between 1970 and 1997¹⁸. We get output data from BEA and employment data from CPS. The estimated model is of the following form:

¹⁸We stop at 1997 as the change in the industry classification standard from SIC to NAICS makes state-level production data comparison unreliable before and after. Since a large part of the improvements in trade secret protection and growth in PBS employment happens in pre-1997, we focus on that interval. Lastly, some of the states only have industry-level employment data after 1977 in CPS. These states' employment, before 1977, is given as an aggregate of 2-4 states. In the end, the analysis is carried through an unbalanced panel with 1177 observations.

Figure 4: Employment Trends in Multiple Industry Groups (1972-2017) In the left panel, solid line includes employment in U.S. Census 1990 3-digit industry groups 721: Advertising, 732: Computer and data processing services, 841: Legal services, 882: Engineering, architectural, and surveying services, 890: Accounting, auditing, and bookkeeping services, 891: Research, development, and testing services, 892: Management and public relations services calculated from Current Population Survey. Total Non-Farm Employment and Services Employment series are from the Current Employment Survey. In the panel on the right, the solid line includes employment in U.S. Census 1990 3-digit industry groups 721, 732, 882, 891. The dashed line includes the employment in the U.S. Census 1990 3-digit industry groups 841 and 890. The dotted line includes employment in the U.S. Census 1990 3-digit industry groups 410: Trucking service, 411: Warehousing and storage, 471: Sanitary services, 722: Services to dwellings and other buildings, 731: Personnel supply services, 740: Detective and protective services. The dashed-dotted line subtracts PBS employment calculated from CPS from Professional and Business Services employment series from CES. CES does not include more detailed industry employment statistics before 1990.



$$y_{it} = \beta x_{it} + \alpha_i + \gamma_t + \epsilon_{it} \quad (1)$$

where x_{it} is the vector of observable explanatory variables including the trade secret protection, α_i is the state fixed-effect and γ_t is the year fixed-effect. We use Auto Correlation consistent White standard errors to account for possible correlation in ϵ_{it} across time. Table 2 gives panel regression results for different model specifications where the dependent variable is the logarithm of employment in PBS sectors with no occupational code for trade secret protection¹⁹. All specifications give a positive relationship between trade secret protection and PBS employment. Moreover, except for the case where only time fixed-effects are excluded, the relation is significant at the 5% level. The full model in (4) suggests an increase in the trade secret protection index from 0 to 1 is associated with a 24% increase in the PBS employment, which is also an economically significant amount.

To make sure what we are capturing is actually the association with trade secret protection and not some other variable that is correlated with trade secret protection but is relevant for all business services, we run additional placebo regressions. In Table 3, we present panel data regression results with controls and both state and year fixed effects where the dependent variable is log PBS employment except for legal and accounting services, log employment in non-professional business services and log employment in legal and accounting services. As predicted by our mechanism, the association between trade secret protection and other business services are both smaller in magnitude and are not significant at 10% level.

To ensure these results are not driven by the particulars of how the trade secret protection index is constructed, we also run differences-in-differences regressions where the index is replaced with a dummy variable that takes the value 1 if the state has implemented UTSA at the given year. As shown in Table 4, the results are both qualitatively and quantitatively similar across these two specifications²⁰.

At this point, it is hard to make causal claims. There is an obvious simultaneity problem; it is conceivable that a larger PBS sector makes it a more pressing issue to have better trade secret protection. To deal with these types of endogeneity issues we use a regression discontinuity design around the adoption of UTSA, as in Jeffers (2018). We run regressions of the following form:

¹⁹Exact contents of sector groups used can be found in the note for Figure 4.

²⁰The adoption of UTSA corresponds to an increase of 0.43 in the index, averaging across states.

Table 2: Panel regressions, PBS Employment

<i>Dependent variable: log(PBS Employment)</i>				
	No FE	Year FE	State FE	State+Year FE
	(1)	(2)	(3)	(4)
TSP	0.294** (0.123)	0.333** (0.136)	0.140 (0.104)	0.237** (0.098)
log(TGDP)		-0.064 (0.315)	1.335** (0.533)	0.823 (0.514)
log(EMP)		0.326 (0.649)	0.019 (0.681)	0.730 (0.703)
log(POP)		0.233 (0.435)	-1.209** (0.560)	-1.006* (0.532)
log(ServEMP)		-0.826 (0.711)	2.600*** (0.980)	1.178 (0.902)
log(ManufGDP)		0.051 (0.086)	-0.270* (0.149)	-0.145 (0.137)
log(ServGDP)		1.380*** (0.488)	-1.064 (0.753)	-0.187 (0.661)

Note:

*p<0.1; **p<0.05; ***p<0.01

Table 3: Panel regressions, State+Year Fixed Effect

	<i>Dependent variable:</i>		
	log(PBS)	log(NPBS)	log(LegAcc)
	(1)	(2)	(3)
TSP	0.237** (0.098)	0.073 (0.064)	0.080 (0.092)
log(TGDP)	0.823 (0.514)	0.353 (0.252)	0.247 (0.343)
log(EMP)	0.730 (0.703)	-0.207 (0.455)	-0.264 (0.837)
log(POP)	-1.006* (0.532)	0.828** (0.332)	0.445 (0.673)
log(ServEMP)	1.178 (0.902)	-0.200 (0.525)	0.022 (0.766)
log(ManufGDP)	-0.145 (0.137)	-0.047 (0.097)	-0.056 (0.135)
log(ServGDP)	-0.187 (0.661)	0.039 (0.384)	0.285 (0.349)

Note: *p<0.1; **p<0.05; ***p<0.01

Table 4: DID, State and Year Fixed Effect

	<i>Dependent variable:</i>		
	PBS	NPBS	LegAcc
	(1)	(2)	(3)
TSA_D	0.096** (0.046)	0.048 (0.030)	0.062 (0.050)
log(TGDP)	0.794 (0.510)	0.344 (0.250)	0.239 (0.337)
log(EMP)	0.743 (0.708)	-0.206 (0.452)	-0.265 (0.836)
log(POP)	-1.008* (0.538)	0.824** (0.331)	0.438 (0.668)
log(ServEMP)	1.187 (0.909)	-0.202 (0.523)	0.019 (0.765)
log(ManufGDP)	-0.145 (0.137)	-0.051 (0.097)	-0.062 (0.134)
log(ServGDP)	-0.152 (0.658)	0.056 (0.382)	0.305 (0.343)

Note: *p<0.1; **p<0.05; ***p<0.01

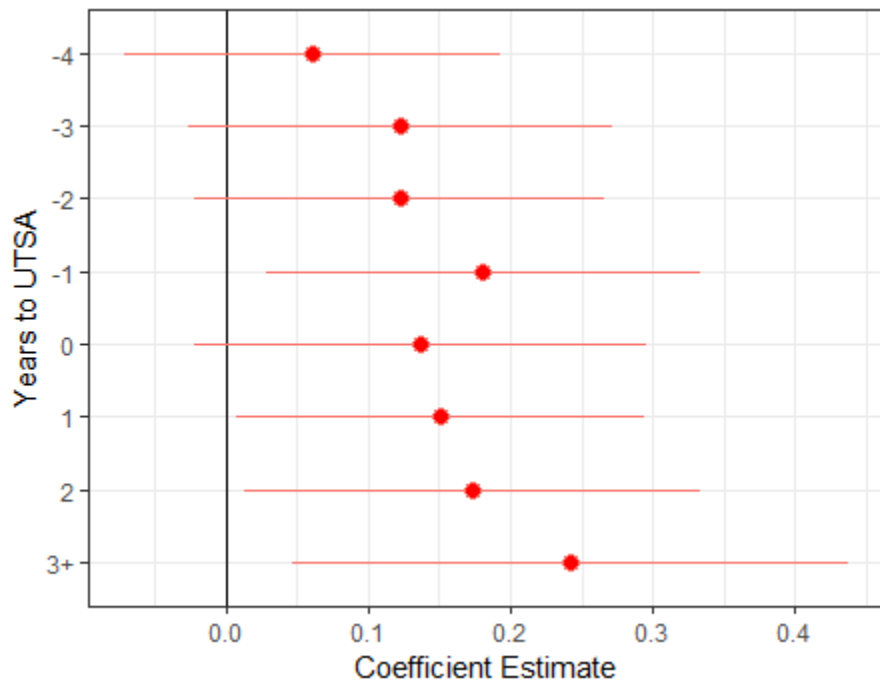


Figure 5: Regression Estimates From Equation 2

$$y_{it} = \sum_k \beta_k (1_{k\text{-years-to-treatment}}) + \alpha_i + \gamma_t + \epsilon_{it} \quad (2)$$

If the law change responds to growth in the PBS sector, we should see a significant difference in the PBS employment of states that have two years and five years to adopting UTSA. Otherwise, there should only be a difference after the adoption of the UTSA, not before. Figure 3 plots the coefficient estimates for β_k with the associated confidence bands, where the intercept corresponds to PBS employment five years prior to the adoption of UTSA.

With the exception of the states that are one year prior to adopting (which could be explained with the anticipation of adoption), the regression confirms our hypothesis. While those states who have not yet adopted UTSA look similar, those who adopted the UTSA have significantly larger PBS employment.

As a further check, we apply the Cox Proportional Hazard model following Png (2017a) to predict the adoption date of UTSA. To be continued.

4 Static Model

4.1 Environment

Preferences

We have a Lucas (1978) type one-period economy with a measure one of risk-neutral agents. There is a single consumption good. Agents maximize their expected income.

Endowments

Agents are characterized by three skill levels: basic, professional S_i and managerial ζ_i . Basic labor skill is identical for everyone while professional and managerial skill is distributed with cdf $F_S(\cdot)$ and $F_\zeta(\cdot)$. Agent i chooses whether to be endowed with one efficiency unit of basic labor (to be called as laborers), S_i efficiency units of skilled labor (to be called as professionals) or a managerial technology $G_{\zeta_i}(\cdot)$ (to be called as managers). That is, agents' skill levels do not affect their income if they become laborers but it potentially affects their income if they choose to be professionals or managers. Both laborers and professionals supply their unit of labor inelastically.

Technology

There are two types of technology in the economy: intermediation technology and production technology.

Each manager i is endowed with a stochastic CES-in-CES production technology that depends on their managerial skill ζ_i and uses basic labor, hired skilled labor and rented skilled labor²¹. The need for skilled labor is stochastic, that is, with probability π , managers enjoy high productivity ι_H and with probability $1 - \pi$, the productivity becomes low ι_L for skilled labor²². Production of a manager with productivity realization ι , skill level ζ , L units of basic labor, S_H units of hired professional labor and S_R units of rented professional labor

²¹Here, instead of treating hired and skilled labor as perfect substitutes, we allow them to have an arbitrary degree of substitutability. In management science literature, rented professionals are usually believed to bring an industry-wide experience and provide an outside perspective that the firm does not have. On the other hand, those who are directly hired know the organization of the firm better and can help rented professionals in their job. Therefore, there is a certain degree of complementarity between them. (Put citation)

²²Stochasticity here is a static reduced form representation of infrequent demand over a longer time horizon. The key point is that hired skilled labor cannot be adjusted while rented skilled labor can be. This could also be generalized to a more complicated productivity shock distribution.

becomes:

$$G_\zeta(L, S_H, S_R) = \zeta \left(L^\psi + \iota(\alpha S_H^\rho + (1 - \alpha)S_R^\rho)^{\psi/\rho} \right)^{\phi/\psi} \quad (3)$$

where $\phi \in (0, 1)$ determines the ‘span of control’ of each manager, i.e. returns to scale for the whole production. $\rho < 1$ and $\psi \in (0, 1)$ determine elasticity of substitution between inputs while $\alpha \in (0, 1)$ and $\iota > 0$ determine output shares of each input.

Managers can only make their hiring decisions before the productivity shock realizes. However, they can choose how much skilled-labor to rent after the realization of the shock. Therefore, renting provides managers with flexibility in terms of production levels but comes at a higher price than hiring. Lastly, productivity shocks are *i.i.d.* across managers, thus there is no aggregate risk.

Firms with intermediation technology hire professional labor and rent to managers with production technology after managers observe their productivity realization. Moreover, intermediate firms provide professionals they hire with a general experience, which they would not be able to access if they were to be directly hired by managers. Each unit of labor that is rented to managers costs an extra $\kappa - 1$ unit of time to the renting firm, on top of the market wage offered. The intermediation technology has constant returns to scale, therefore its ownership is irrelevant. $\kappa > 1$ is a reduced form representation for transaction costs related to renting which includes the expected loss due to trade secret violation. The intermediated sector represents Professional Business Services in our setting.

Occupational Decision

Each agent i chooses an occupation, comparing wage income from being a laborer w_L , wage income from being a professional $S_i w_S$ and expected profits from opening up a firm Π_{ζ_i} .

Manager’s Problem

Managers with skill level ζ choose how much basic labor L and skilled labor S_H to hire and how much professional labor to rent after high productivity shock $\overline{S_R}$ and low productivity shock $\underline{S_R}$ to solve:

$$\begin{aligned}
\Pi(\zeta) = \max_{\{L, S_H, \overline{S_R}, \underline{S_R}\} \geq 0} & \pi \zeta \left(L^\psi + \iota_H (\alpha S_H^\rho + (1 - \alpha) \overline{S_R}^\rho)^{\psi/\rho} \right)^{\phi/\psi} \\
& + (1 - \pi) \zeta \left(L^\psi + \iota_L (\alpha S_H^\rho + (1 - \alpha) \underline{S_R}^\rho)^{\psi/\rho} \right)^{\phi/\psi} \\
& - w_L L - w_S S_H - w_S \pi \kappa \overline{S_R} - w_S (1 - \pi) \kappa \underline{S_R}
\end{aligned} \tag{4}$$

where w_L and w_S are the unit prices for hiring basic and professional labor²³.

Equilibrium

Definition A competitive equilibrium would be a set of prices $\{w_L, w_S\} \in R_+^2$, occupational decisions $\{O_i\}_{i=0}^1 \in \{L, S, M\}$ and allocations $\{L_i, S_{H,i}, \overline{S_{R,i}}, \underline{S_{R,i}}, \Pi_i, \}_{i|O_i=M}$ such that

- (i) Each agent i chooses her occupation and each manager i choose inputs of production optimally, given prices.
- (ii) Prices clear labor markets.

4.2 Solution

We will restrict attention to a specific case where we have perfect sorting to occupations in equilibrium.

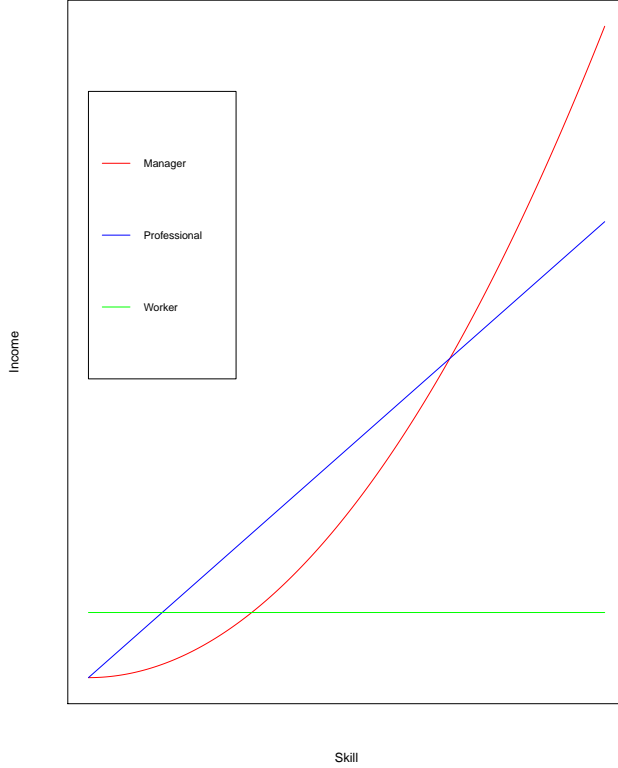
Lemma 1. *Assume $S_i = \zeta_i = i$ and $\text{supp}(F) \cap (-\infty, 0] \neq \emptyset$. In any competitive equilibrium, $\exists l$ and $u \in (l, \infty)$ such that those with skill level in $(-\infty, l]$ become laborers, $(l, u]$ professionals and (u, ∞) managers.*

Proof. Fix $\{w_L, w_S\}$. Income from being a laborer is flat with respect to skill while the income of a professional is a linearly increasing function of skill. Profit of a manager, on the other hand, is a convex function of skill²⁴. We know at 0 skill level, being a professional or a manager generates zero income. There has to be both laborers and professionals in

²³The managers in the model are both owners of the production and they supply a managerial input into the production. Therefore, the profits coming from the technologies could possibly be interpreted as profits or compensation of managers. We interpret the profits as managerial compensation. This can be justified in a slightly more complicated model with identical technology owners with no bargaining power who hire managers to operate their technologies for them.

²⁴A manager with twice the ability produces twice the output, conditional on using the same amount of inputs. Moreover, a manager of higher ability would like to employ more inputs, further increasing the managerial income.

the economy since the marginal product of both go to infinity as their number goes to 0. Moreover, for there to exist wage rates, there must exist some managers as well. Therefore, the only possible scenario²⁵ that can happen in equilibrium is as the one in Figure 4.2. \square



This simplifies the equilibrium, since the occupational decisions of a continuum of agents can be represented with two threshold skill levels. Then, equilibrium would be a set of prices $\{w_L, w_S\}$, occupation thresholds l, u and set of allocations $\{L_i, S_{H,i}, \overline{S_{R,i}}, \underline{S_{R,i}}, \Pi_i\}_{\{i|O_i=M\}}$ such that allocations solve (4), markets for basic and professional labor clear

$$\begin{aligned} \int_u^\infty L_i dF(i) &= F(l) \\ \int_u^\infty \left(S_{H,i} + \pi \overline{S_{R,i}} + (1 - \pi) \underline{S_{R,i}} \right) dF(i) &= \int_l^u i dF(i) \end{aligned} \tag{5}$$

an agent with skill l is just indifferent between being a laborer and a professional

²⁵If not for the assumption that agents at the bottom have zero managerial/professional skill, we could have a double-crossing between managers and professionals. In that case, a possible scenario would be where lower skill agents also choose to become managers.

$$w_L = lw_S \quad (6)$$

and an agent with skill u is just indifferent between being a professional and a manager.

$$uw_S = \Pi_u \quad (7)$$

4.3 Characterizing the Solution

For ease of exposition and to be able to get analytic solutions, we will assume a more specific form of production function to replace (21)²⁶

$$F_\zeta(L, S_H, S_R) = \zeta \left(L^\psi + \iota S_H^\rho S_R^{\phi-\sigma} \right) \quad (8)$$

Then, the manager's problem can be simplified as

$$\begin{aligned} \Pi(\zeta) = \max_{\{L, S_H, \overline{S}_R, \underline{S}_R\} \geq 0} & \zeta \left(L^\psi + \pi \iota_H S_H^\sigma \overline{S}_R^{\phi-\sigma} + (1-\pi) \iota_L S_H^\sigma \underline{S}_R^{\phi-\sigma} \right) \\ & - w_L L - w_S S_H - w_S \pi \kappa \overline{S}_R - w_S (1-\pi) \kappa \underline{S}_R \end{aligned} \quad (9)$$

Proposition 1. *Under assumptions that imply perfect sorting into occupations, for a continuous skill distribution F and for the production function given in (8), equilibrium exists and it is unique.*

Proof. See Appendix 8.2. □

We want to analyze how does the economy respond to an unexpected change in the cost of renting κ in the short, medium and long run. Mechanically, we will check how do the equilibrium allocations and prices change with changes in κ . The short run will be defined as the situation where both l and u are fixed, the medium run will be where l is fixed but u can adjust and long run will be where both l and u can adjust²⁷.

²⁶Here, we rule out any substitution/complementarity between basic and skilled labor and assumed a very specific substitution/complementarity between hired and rented skilled labor. Former has implication for our qualitative results while the latter does not. The qualitative results would still carry through to a scenario where ψ is sufficiently close to 1 in (21).

²⁷We claim l takes longer to adjust than u does. While changes in u are entrepreneurial decisions of skilled agents, large changes in l can only happen through educational decisions of the college-age population.

Proposition 2. *Holding l and u fixed, as cost of intermediation κ goes down,*

- (1) *Fraction of the professionals who are rented ($\int(\pi\bar{S}_R^i + (1-\pi)\underline{S}_R^i)di / \int(\pi\bar{S}_R^i + (1-\pi)\underline{S}_R^i + S_H^i)di$) increases,*
- (2) *Wages of professionals w_S and profits of each manager Π_i increases while wages of laborers w_L stays constant,*
- (3) *(Ex-ante) income dispersion within professionals ($V(w_S\zeta)$) and within managers ($V(\Pi(\zeta))$) increases.*

Proof. (1) The first part of the theorem comes directly from the first order conditions of the managers given in (36). As κ goes down, renting becomes cheaper relative to hiring and each manager prefer to rent more. Since the fraction of hired versus rented professionals is constant across managers, the overall fraction of professionals rented goes up as well.

(2) The next two parts requires characterizing the solution further. First order conditions and the market clearing conditions give analytical expressions for w_L , w_S and Π :

$$\begin{aligned}
w_L &= \psi \left(\frac{f_M^L}{F(l)} \right)^{1-\psi} \\
w_S &= \sigma \iota_c^{1+\sigma-\phi} \left[\left(\frac{\phi-\sigma}{\sigma\kappa} \right)^{\frac{\phi-\sigma}{1-\phi}} + \left(\frac{\phi-\sigma}{\sigma\kappa} \right)^{\frac{1-\sigma}{1-\phi}} \right]^{1-\phi} \left(\frac{f_M^S}{f_S} \right)^{1-\phi} \\
\Pi &= \zeta^{\frac{1}{1-\phi}} (1-\phi) \left(\frac{1}{\kappa} \right)^{\phi-\sigma} \iota_c^{1+\sigma-\phi} \left(\frac{\phi-\sigma}{\sigma} \right)^{\phi-\sigma} \left[1 + \frac{\phi-\sigma}{\sigma\kappa} \right]^{-\phi} \left(\frac{f_S}{f_M^S} \right)^\phi \\
&\quad + \zeta^{\frac{1}{1-\psi}} (1-\psi) \left(\frac{F(l)}{f_M^L} \right)^\psi
\end{aligned} \tag{10}$$

It can directly be seen w_L does not depend on κ and w_S is decreasing in κ while simple differentiation shows Π is increasing in κ for $\kappa \geq 1$.

(3) This part is a corollary to (2). Since w_S increases, the increase in the total wage income of professional with skill ζ (ζw_S) is larger for professionals with larger ζ . Similarly, for managers, increase in profits with respect to a decrease in κ ($-\frac{\partial \Pi}{\partial \kappa}$) is an increasing and convex function of ζ . Therefore, income dispersion increases for both professionals and managers. \square

Proposition 2 suggests, after a law change that improves trade secret protection, we would expect a growth in PBS sector employment and an increase in overall and within-group wage inequality in the short run²⁸. This change, however, would also affect incentives of high-skill

²⁸For managers, we think the relevant object is the ex-ante income. Given that the uncertainty is a

individuals to be employed as professionals instead of starting their own businesses. This would change the threshold skill level u in the economy in the medium run:

Proposition 3. *Holding l constant, as the cost of intermediation κ goes down,*

- (1) *The fraction of the professionals who are rented increases,*
- (2) *The number of professionals increases while the number of managers decreases,*
- (3) *The wages of laborers w_L go down while the wages of professionals w_S and profits of old managers Π go up.*
- (4) *(Ex-ante) income dispersion within professionals increases. The income distribution within managers has a higher lower bound with a fatter tail.*

Proof. (1) Same as Proposition 2.

(2) When u is also endogenous, we have one additional equation: agent with skill level u has to be indifferent between being a professional and a manager. Substituting optimality conditions of the managers and market clearing conditions into the indifference condition, we reduce the system into a single equation and single unknown u :

$$\begin{aligned} \sigma \left(\frac{\phi - \sigma}{\sigma} \right)^{\phi - \sigma} \left[1 + \frac{\phi - \sigma}{\sigma \kappa} \right]^{1 - \phi} \left(\frac{f_M^S}{f_S} \right)^{1 - \phi} &= u^{\frac{\phi}{1 - \phi}} (1 - \phi) \left(\frac{\phi - \sigma}{\sigma} \right)^{\phi - \sigma} \left[1 + \frac{\phi - \sigma}{\sigma \kappa} \right]^{-\phi} \left(\frac{f_S}{f_M^S} \right)^{\phi} \\ &+ u^{\frac{\psi}{1 - \psi}} (1 - \psi) \left(\frac{F(l)}{f_M^L} \right)^{\psi} \kappa^{\phi - \sigma} \end{aligned} \quad (11)$$

LHS is decreasing with u while RHS is increasing. Moreover, LHS decreases with κ and RHS increases. Therefore, $u^*(\kappa)$ is a decreasing function of κ . Therefore, following a decrease in κ , u goes up.

(3) Rest of the equilibrium conditions are identical to the ones in (10). When u goes up, the supply of professionals (through f_S) goes up while the supply of managers (through f_M^L and f_M^S) go down. It is immediate that w_L goes down and Π goes up. For w_S , however, decrease in f_M^S and increase in f_S moves in a direction that counteracts the initial increase in w_S due to the direct effect of decreasing κ . On the other hand, for u to go up in equilibrium, those at the margin who used to be managers must be choosing to switch to being professionals. Therefore, it must be the case that overall effect on w_S is positive.

(4) Follows the reasoning in Proposition 2. One caveat is now some agents switch from

reduced form representation of infrequent demand over the long run, we should expect fluctuations to cancel each other in a longer horizon. Unless otherwise stated, all analysis regarding managerial income will be for ex-ante expected income.

being managers to professionals. Since those who switch have a higher skill level than existing professionals, there would be an additional effect increasing the observed wage dispersion among the professionals. For the managers, since the lowest skilled ones have left, there is a counteracting force that decreases the dispersion. However, for those who continue to be managers, income dispersion increases similar to Proposition 2. Therefore, we can conclude the tail of the income distribution becomes fatter. \square

Proposition 3 suggests, after the initial increase in w_S and Π suggested by Proposition 2, some agents prefer to be employed as professionals over having their own firm. This change decreases the demand for laborers and counteracts some of the initial increase in demand for professionals. Thus, we would observe a decrease in w_S (although it would still be higher than the value before the law change) and w_L in the medium run. Then, individuals would be more willing to be employed as professionals rather than as laborers in the long run. This would change the threshold skill level l in the economy in the long run, creating upward pressure on l .

While all the equilibrium conditions from the previous case remain, full equilibrium would have one additional variable l and one additional indifference condition for agent with skill level l :

$$\psi \left[\frac{f_M^L}{F(L)} \right]^{1-\psi} = u \sigma \iota_c^{1+\sigma-\phi} \left(\frac{\phi - \sigma}{\sigma \kappa} \right)^{\phi-\sigma} \left[1 + \frac{\phi - \sigma}{\sigma \kappa} \right]^{1-\phi} \left[\frac{f_M^S}{f_S} \right]^{1-\psi} \quad (12)$$

Therefore, the equilibrium conditions can be reduced to two unknowns l, u and two equations : (11) and (12). Depending on the size of the change in l relative to the size of the change in u , w_L can decrease or increase with κ . We have no simple analytical expression for how the equilibrium objects would move with respect to κ as of now. Therefore, we will proceed with numerical analysis for the full static model.

4.4 Numerical Analysis

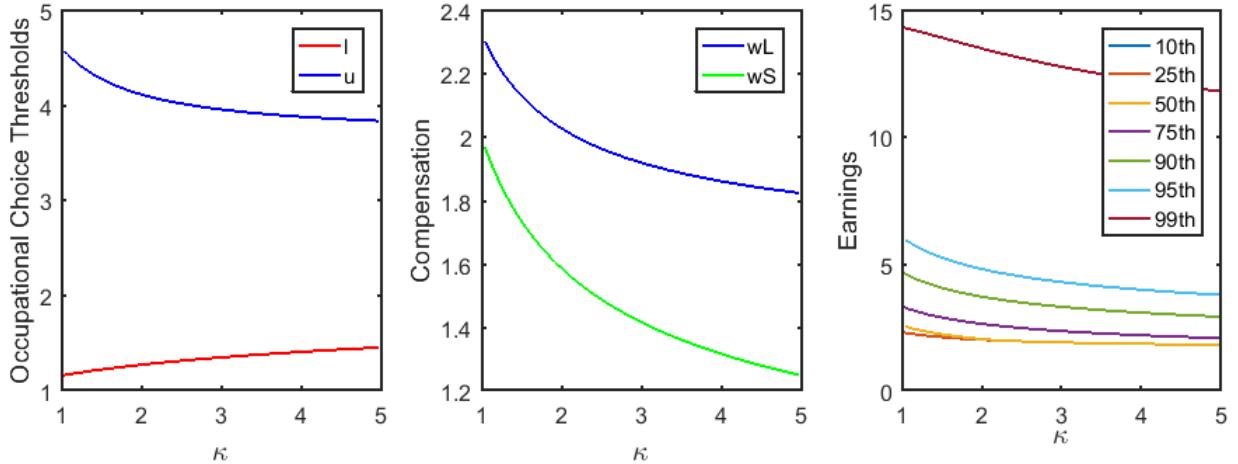
The purpose is to see how allocations and prices move with respect to κ . Skill distribution is assumed to be Pareto where c.d.f. is $F(x) = 1 - (x_0/x)^\gamma$. The values of the parameters that are used in the numerical exercise are given in Table 5.

Given the parameter values, how the equilibrium changes with κ is given in Figure 4.4. Figure 4.4a shows l decreases and u increases with κ . Figure 4.4b shows the compensation for efficiency unit of unskilled (w_L) and skilled labor (w_S) increases while the latter increases

Table 5: Parameter Values for Numerical Exercise

Production Function						Skill Distribution	
ϕ	ψ	π	σ	ι_H	ι_L	γ	x_0
0.4	0.4	0.5	0.2	5	1	2	1

Figure 6: Comparative Statics for κ



at a faster rate²⁹. Figure 4.4c shows how do the percentiles of earnings distribution changes with κ . Although the income of each group increases, the increase in the top is much larger than the increase in the bottom.

Comparative statics exercise has multiple implications. First, as κ goes down, the number of laborers $F(l)$ and managers $(1 - F(u))$ go down. The compensation for all occupational groups ends up increasing. Lastly, dispersion of income increases although underlying skill distribution did not change. The increase in dispersion happens both between and within occupational groups.

To summarize, the full model gives all the labor market trends we have discussed in the introduction as a result. PBS sector grows, entrepreneurial activity slows down and income inequality grows mainly at the top half. We will evaluate the quantitative significance of each of these effects in a more general dynamic model.

²⁹The compensation of professionals is $w_S\zeta$ where $\zeta > 1$. Therefore it is possible that $w_S < w_L$.

5 Dynamic Model

5.1 Environment

Here, we will extend the static model to a dynamic overlapping generations (OLG) environment. Measure one of agents with heterogeneous skills live for two periods (young and old) and maximize expectation of the present value of their income. There are no assets in the economy.

Agents start their lives by choosing whether to be employed as laborers or professionals. Conditional on being a professional, they can choose whether to be hired versus rented. Being rented improves human capital accumulation more, thus compensation need not be equal for hired and rented individuals.

Those who chose to be laborers in the first period are bound to continue as laborers in the second period. Those who were employed as professionals, however, can choose between being hired/rented as professionals and being managers and operating their own technology.

Thus, each period starts with young and old making simultaneous occupational decisions. Then production occurs and lastly, human capital accumulation is realized.

States

Individual state is the skill level ζ while the aggregate state is the distribution of skills of the old who were previously employed as professionals F_O . For old who were laborers when they were young, ζ becomes irrelevant as they are bound to continue as laborers. Here, wealth does not affect any decisions, therefore its distribution is irrelevant.

Occupational Decision

Each young agent with skill level ζ chooses an occupation, comparing value of being a laborer $V_Y^L(\zeta, F_O)$, being hired as a professional $V_Y^{SH}(\zeta, F_O)$ and being rented as a professional $V_Y^{SR}(\zeta, F_O)$. Similarly, each old agent (who were not laborers in the previous period) with skill level ζ chooses an occupation, comparing value of being hired as a professional $V_O^{SH}(\zeta, F_O)$, being rented as a professional $V_O^{SR}(\zeta, F_O)$ and being a manager $V_O^M(\zeta, F_O)$. Denoting wage of laborers with w_L , the wage of professional hires with w_S^H and wage of rented professionals with w_S^R , we can write the value functions for the old as

$$\begin{aligned}
V_O^{SH}(\zeta, F_O) &= \zeta w_S^H(F_O) \\
V_O^{SR}(\zeta, F_O) &= \zeta w_S^R(F_O) \\
V_O^M(\zeta, F_O) &= \Pi(\zeta, F_O) \\
V_O^L(F_O) &= w_L(F_O)
\end{aligned} \tag{13}$$

where $\Pi(\zeta, F_O)$ is the value of the manager's problem:

$$\begin{aligned}
\Pi(\zeta, F_O) = \max_{\{L, S_H, \overline{S_R}, \underline{S_R}\} \geq 0} & \pi \zeta \left(L^\psi + \iota_H (\alpha S_H^\rho + (1 - \alpha) \overline{S_R}^\rho)^{\psi/\rho} \right)^{\phi/\psi} \\
& + (1 - \pi) \zeta \left(L^\psi + \iota_L (\alpha S_H^\rho + (1 - \alpha) \underline{S_R}^\rho)^{\psi/\rho} \right)^{\phi/\psi} \\
& - w_L(F_O)L - w_S^H(F_O)S_H - w_S^R(F_O)\pi\kappa\overline{S_R} - w_S^R(F_O)(1 - \pi)\kappa\underline{S_R}
\end{aligned} \tag{14}$$

We can denote the expected option value of being a professional in the first period with

$$V^S(\zeta, F_O) = \max(V_O^{SH}(\zeta, F_O), V_O^{SR}(\zeta, F_O), E[V_O^M(\zeta, F_O)]) \tag{15}$$

Value functions for the young become

$$\begin{aligned}
V_Y^{SH}(\zeta, F_O) &= \zeta w_S^H(F_O) + \beta V^S(\zeta', F'_O) \\
V_Y^{SR}(\zeta, F_O) &= \zeta w_S^R(F_O) + \beta V^S(\zeta'', F'_O) \\
V_Y^L(\zeta, F_O) &= w_L(F_O) + \beta V_O^L(F'_O)
\end{aligned} \tag{16}$$

where $\zeta' < \zeta''$. This represents the additional human capital accumulation for those professionals working in PBS firms. Therefore, we can write down the problem of a young agent as

$$\begin{aligned}
V_Y(\zeta, F_O) &= \max\{\zeta w_S^H(F_O) + \beta V^S(\zeta', F'_O), \zeta w_S^R(F_O) + \beta V^S(\zeta'', F'_O), w_L(F_O) + \beta V_O^L(F'_O)\} \\
& \text{s.t.} \\
& F'_O = G(F_O) \\
& \zeta' = h_1(\zeta) \\
& \zeta'' = h_2(\zeta)
\end{aligned} \tag{17}$$

Equilibrium

Definition A rational expectations recursive competitive equilibrium would be a set of prices $\{w_L(F_O), w_S^H(F_O), w_S^R(F_O)\} \in R_+^3$, occupational decisions of the young $\{O_i^Y(F_O, \zeta_i)\}_{i=0}^1 \in \{L, S_H, S_R\}$, occupational decisions of the skilled old $\{O_j^O(F_O, \zeta_i)\}_{j|O_j^Y \in \{S_H, S_R\}} \in \{M, S_H, S_R\}$, allocations $\{L_i(F_O, \zeta_i), S_{H,i}(F_O, \zeta_i), \overline{S_{R,i}}(F_O, \zeta_i), \underline{S_{R,i}}(F_O, \zeta_i), \Pi_i(F_O, \zeta_i), \}_{i|O_i^O=M}$ and evolution of skill distribution of the old $G(F_O)$ such that

- (i) Each cohort chooses occupations and managers choose inputs of production optimally, given prices.
- (ii) Prices clear labor markets.
- (iii) Agents' beliefs on how the skill distribution will evolve coincide with the actual evolution.

5.2 Solution

We will first restrict attention to the steady state of the economy where the distribution of talent stays constant over time. In that case, the problem of the manager would simplify to the one in the static model. We further assume $h_2(\zeta) > h_1(\zeta) \forall \zeta$ and $h_2(\zeta) - h_1(\zeta)$ is strictly increasing in ζ . In other words, more talented individuals benefit more from the additional human capital accumulation in the PBS sector. Lastly, we will assume that the optimal number of rented individuals is always lower than the measure of young.

Lemma 2. *Assume $S_i = \zeta_i = i$, $\text{supp}(F) \cap (-\infty, 0] \neq \emptyset$ and $h_1(0) = h_2(0) = 0$. In any competitive equilibrium, $\exists l_Y, u_Y$ such that young with skill level in $(-\infty, l_Y]$ become laborers, $(l_Y, u_Y]$ hired professionals and (u_Y, ∞) rented professionals. Moreover, in any competitive equilibrium, $\exists u_O$ such that old professionals with skill level in $(l_Y, u_O]$ becomes hired professionals and (u_O, ∞) managers.*

Proof. Given that being rented has the extra advantage of faster human capital accumulation, $w_S^H > w_S^R$. Since old does not care about human capital accumulation, all the rented jobs will be filled by the young.

Fix $\{w_L, w_S^H, w_S^R\}$ where $w_S^R > w_S^H$. For an old professional, income from remaining a professional is a linearly increasing function of skill. Profit from being a manager, on the other hand, is a convex function of skill. Since only old can be managers, u_O has to exist.

For a young agent, income from being a laborer is constant with skill while income from being a professional is an increasing function of skill. Moreover, the marginal product of labor goes to infinity as the number of laborers go to zero. Therefore, l_Y must exist. Furthermore,

the additional benefit of being rented is an increasing function of skill. Also, the marginal product of rented professionals go to infinity as the number of rented professionals go to zero. Thus, u_Y must exist. \square

With Lemma 2, the steady state equilibrium is reduced to a set of prices $\{w_L, w_S^H, w_S^R\} \in R_+^3$, occupational decision thresholds $l_Y, u_Y, u_O \in \text{supp}(F)$, and decisions of managers $\{L_i(\zeta_i), S_{H,i}(\zeta_i), \overline{S_{R,i}}(\zeta_i), \underline{S_{R,i}}(\zeta_i), \Pi_i(\zeta_i), \}_{i|\zeta_i > u_O}$.

The equilibrium conditions will be the first order conditions of the managers, which are identical to the ones in the static model, market clearing for basic, hired professional and renter professional labor

$$\begin{aligned} \int_{u_O}^{\infty} L_i dF_O(i) &= 2F(l_Y) \\ \int_{u_O}^{\infty} S_{H,i} dF_O(i) &= \int_{l_Y}^{u_Y} idF(i) + \int_{-\infty}^{u_O} idF_O(i) \\ \int_{u_O}^{\infty} \left(\pi \overline{S_{R,i}} + (1 - \pi) \underline{S_{R,i}} \right) dF_O(i) &= \int_{u_Y}^{\infty} idF(i) \end{aligned} \quad (18)$$

occupational indifference conditions for the young

$$\begin{aligned} V_Y^L(l_Y) &= V_Y^{SH}(l_Y) \\ V_Y^{SH}(u_Y) &= V_Y^{SR}(u_Y) \end{aligned} \quad (19)$$

and occupational indifference condition for the old:

$$V_O^{SH}(u_O) = \Pi(u_O) \quad (20)$$

To be continued.

6 Model Estimation

In this part, we assume initial skills have a Pareto distribution with cdf $F(x) = 1 - (x_0/x)^\gamma$. Moreover, skill accumulation functions in professional firms have multiplicative forms: $h_1(\xi) = \nu_1 \xi$ and $h_2(\xi) = \nu_2 \xi$. The key parameters of the model are those from the production function $\pi, \nu_H, \nu_L, \alpha, \psi, \phi, \rho$, those from the initial skill distribution x_0, γ , those from the skill accumulation function ν_1, ν_2 , additional cost of renting κ and the discount factor β .

We plan to estimate the production function parameters outside the model using firm-level panel data. We will proceed by calibrating β consistent with interest rates. Lastly, we will estimate parameters regarding skill distribution and accumulation and information friction associated with renting using the simulated method of moments, matching moments on wage dynamics, firm size distribution, size of the PBS sector and wage dynamics of professionals in the U.S. economy.

6.1 Production Function Estimation

We are in the process of requesting confidential U.S. Census firm data to estimate the production function. This will be the first paper that attempts to estimate a production function where outsourced inputs are modeled explicitly. All the methodology described here is conditional on being able to get the relevant Census data.

For this part, Census data will provide (1) micro level panel data necessary for estimation of the production function outside the model and (2) moments to be matched for estimation of the skill distribution of individuals. We will specifically need the latest panels (2014-2018) of the Annual Survey of Manufacturers (ASM) and the Services Annual Survey (SAS) and the latest Economic Census (EC 2012).

The production function to be estimated has the CES-in-CES form:

$$G_{\zeta}(L, S_H, S_R) = \zeta \left(L^{\psi} + \iota(\alpha S_H^{\rho} + (1 - \alpha) S_R^{\rho})^{\psi/\rho} \right)^{\phi/\psi} \quad (21)$$

ζ denotes skill level of the manager, L denotes low-skill labor, S_H denotes units of hired professional labor and S_R units of rented professional labor. Here, $\phi \in (0, 1)$ determines returns to scale for the whole production, $\rho < 1$ and $\psi \in (0, 1)$ determine elasticity of substitution between inputs while $\alpha \in (0, 1)$ and $\iota > 0$ determine output shares of each input. The model assumes ι follows a random process. The firm chooses L and S_H before observing the realization of ι while the choice of S_R is made after it. This represents the fact that hiring decisions are made long term with large hiring and firing costs while rental decisions can be made for shorter horizons. Thus, we are treating S_R as an adjustable input while L and S_H as unadjustable inputs. We assume ι is drawn from a Bernoulli distribution with possible values of ι_H and ι_L where ι_H is drawn with probability π . The parameters to be estimated here are ψ , α , ρ , ϕ , ι_H , ι_L and π .

In ASM, we will use Production workers annual wages (PAYANNPW) for L , annual payroll (PAYANN) for $L + S_H$, and PCHPBS (as defined in $PCHPBS = PCHPRTE +$

$PCHADVT+PCHADPR$) for S_R ³⁰. We will use Value added (VALADD) for $G_\zeta(L, S_H, S_R)$ ³¹.

In SAS, there are two differences compared to ASM. Firstly, there is no direct variable for value added. Instead, we will construct it by subtracting Expensed purchases of other materials, parts and supplies (EXPMPS), Purchased fuels (CSTFU) and Purchased electricity (CSTELEC) from Revenue. Secondly, expenditure on labor is not divided into low skill and high skill to separately identify L and S_H . Instead, we will use Lasso-based methods to determine the predictors of the ratio of L to S_H in ASM dataset. Then, we will use this predictive model in SAS and construct a new variable similar to PAYANNPW.

After these steps, for a given value of ι , estimation becomes possible using a nested NLS. The estimation will be done in three steps. First, we treat $\iota(\alpha S_H^\rho + (1-\alpha)S_R^\rho)^{\psi/\rho}$ as a residual and estimate the parameters of the nonlinear regression

$$G_\zeta(L, S_H, S_R) = \zeta \left(L^\psi + \epsilon \right)^{\phi/\psi} \quad (22)$$

In the second step, we identify the random process for ι from the equation for ϵ using a similar procedure as ?:

$$\epsilon = \iota(\alpha S_H^\rho + (1-\alpha)S_R^\rho)^{\psi/\rho} \quad (23)$$

After we know which time period corresponds to which realization of ι , we can estimate the equation 23, using an IV regression where we instrument S_R with previous year's value.

To be continued.

7 Extensions

7.1 Firms with Multiple Branches

Many firms, especially large ones, consist of multiple units with some idiosyncrasy in their management. These could be multiple divisions in a single firm (human resources, risk management etc.), multiple product lines or multiple branches in different geographical areas.

³⁰Ideally, we would like to use real variables (hours, real output etc.) for labor inputs. However, we don't have information on professional hours. For comparability, we will use the nominal expenditures in place of each of these inputs. Since we will work with a short panel, we expect fluctuations in labor input prices to be a lot smaller than fluctuations in output prices and capital prices. Thus, using nominal values is not as problematic.

³¹We will deflate VALADD with industry level prices using NBER Productivity Database to minimize the effect of demand shocks in estimation, following Foster et al. (2008).

If the skilled labor demand is not perfectly correlated across these units, allocation could be improved by moving hired professionals to whichever unit that need them the most at the time. Consequently, those firms with multiple units could be expected to benefit less from easier access to renting opportunities compared to firms with a single unit. For ease of exposition, we will call each of these units as branches in a firm.

Environment

The environment is the same as in the previous section, with a single added component. Managers, before observing their productivity realization, can pay a fixed cost Υ and replace their technology with two half technologies with independent productivity realizations. Moreover, managers can move hired skilled labor freely across these two half technologies, after observing their productivity realizations. We allow managers to divide their technology into two instead of expanding into another technology to focus on risk-pooling effects of branching instead of the size effects.

Manager's Problem

We will first write down the problem of a manager who has already paid the fixed cost and switched to two half technologies. In order to distinguish the policy functions, we will use \mathcal{L} and \mathcal{S} for managers with two branches. Lastly, we will restrict attention to Cobb-Douglas with separable laborer production given in (8). Managers with skill level ζ choose how much basic labor \mathcal{L} and skilled labor \mathcal{S}_H to hire, how to allocate skilled labor to two technologies if realizations differ $\tilde{\mathcal{S}}_H, \underline{\mathcal{S}}_H$, how much to rent if both productivities turn out to be high $\overline{\mathcal{S}}_R$, how much to rent if both turn out to be low $\underline{\mathcal{S}}_R$ and how much professional labor to rent if productivities differ $\underline{\underline{\mathcal{S}}}_R, \overline{\overline{\mathcal{S}}}_R$ to solve:

$$\begin{aligned}
\Pi^B(\zeta) = & \max_{\{\mathcal{L}, \mathcal{S}_H, \tilde{\mathcal{S}}_H, \underline{\mathcal{S}}_H, \bar{\mathcal{S}}_R, \bar{\mathcal{S}}_R, \underline{\mathcal{S}}_R, \underline{\mathcal{S}}_R\} \geq 0} \zeta \left[\mathcal{L}^\psi + \pi_H^2 \iota_H \mathcal{S}_H^\sigma \bar{\mathcal{S}}_R^{\phi-\sigma} \right. \\
& \left. + \pi_H \pi_L (\iota_H \tilde{\mathcal{S}}_H^\sigma \bar{\mathcal{S}}_R^{\phi-\sigma} + \iota_L \mathcal{S}_H^\sigma \underline{\mathcal{S}}_R^{\phi-\sigma}) + \pi_L^2 \iota_H \mathcal{S}_H^\sigma \underline{\mathcal{S}}_R^{\phi-\sigma} \right] \\
& - w_L \mathcal{L} - w_S \mathcal{S}_H - w_S \kappa [\pi_H^2 \bar{\mathcal{S}}_R + \pi_H \pi_L (\underline{\mathcal{S}}_R + \bar{\mathcal{S}}_R) + \pi_L^2 \underline{\mathcal{S}}_R] \\
& s.t. \\
& \tilde{\mathcal{S}}_H + \underline{\mathcal{S}}_H = 2\mathcal{S}_H
\end{aligned} \tag{24}$$

where we change the notation for π and $1 - \pi$ with π_H and π_L for reasons that will be apparent shortly. We still impose $\pi_H + \pi_L = 1$. Having two branches would give the manager the flexibility to allocate a larger proportion of hired professionals to the technology with higher productivity. Therefore, we would expect to see $\bar{\mathcal{S}}_R > \bar{\mathcal{S}}_R > \underline{\mathcal{S}}_R > \underline{\mathcal{S}}_R$.

Equilibrium

Definition A competitive equilibrium would be a set of prices $\{w_L, w_S\} \in R_+^2$, occupational decisions $\{O_i\}_{i=0}^1 \in \{L, S, M\}$, branching decisions for managers $\{B_i\}_{i|O_i=M} \in \{1, 2\}$, allocations for managers with a single branch $\{L_i, \mathcal{S}_{H,i}, \bar{\mathcal{S}}_{R,i}, \underline{\mathcal{S}}_{R,i}, \Pi_i\}_{i|O_i=M, B_i=1}$ and allocations for managers with two branches $\{\mathcal{L}_i, \mathcal{S}_{H,i}, \tilde{\mathcal{S}}_{H,i}, \underline{\mathcal{S}}_{H,i}, \bar{\mathcal{S}}_{R,i}, \bar{\mathcal{S}}_{R,i}, \underline{\mathcal{S}}_{R,i}, \underline{\mathcal{S}}_{R,i}\}_{i|O_i=M, B_i=2}$ such that

- (i) Each agent i chooses her occupation and each manager i chooses how many branches to operate and inputs of production optimally, given prices.
- (ii) Prices clear labor markets.

Solution

Lemma 3. *The value of the problem presented in (24) can be written in the following form:*

$$\begin{aligned}
\Pi^B(\zeta) = & \max_{\{\mathcal{L}, \mathcal{S}_H, \bar{\mathcal{S}}_R, \underline{\mathcal{S}}_R\} \geq 0} \zeta \left(\mathcal{L}^\psi + \pi_H \iota_H \mathcal{S}_H^\sigma \bar{\mathcal{S}}_R^{\phi-\sigma} + \pi_L \iota_L \mathcal{S}_H^\sigma \underline{\mathcal{S}}_R^{\phi-\sigma} \right) \\
& - w_L \mathcal{L} - w_S \mathcal{S}_H - w_S \pi_H \kappa \bar{\mathcal{S}}_R - w_S \pi_L \kappa \underline{\mathcal{S}}_R
\end{aligned} \tag{25}$$

which is isomorphic to the original problem in (9) where

$$\pi_H = \pi_H^2 + \pi_H \pi_L \left(\frac{(\iota_H/\iota_L)^{\frac{1}{1-\phi}}}{1 + (\iota_H/\iota_L)^{\frac{1}{1-\phi}}} \right)^{\frac{\phi-\sigma}{1-\phi+\sigma}} \quad \pi_L = \pi_L^2 + \pi_H \pi_L \left(\frac{1}{1 + (\iota_H/\iota_L)^{\frac{1}{1-\phi}}} \right)^{\frac{\phi-\sigma}{1-\phi+\sigma}} \quad (26)$$

Proof. From first order conditions for $\overline{\overline{\mathcal{S}}}_R$, $\overline{\mathcal{S}}_R$, $\underline{\mathcal{S}}_R$ and $\underline{\underline{\mathcal{S}}}_R$, one can derive

$$\tilde{\mathcal{S}}_H \left(\frac{\overline{\overline{\mathcal{S}}}_R}{\overline{\mathcal{S}}_R} \right)^{\frac{1-\phi+\sigma}{\sigma}} = \underline{\mathcal{S}}_H \left(\frac{\underline{\underline{\mathcal{S}}}_R}{\underline{\mathcal{S}}_R} \right)^{\frac{1-\phi+\sigma}{\sigma}} \quad (27)$$

and

$$\left(\frac{\overline{\overline{\mathcal{S}}}_R}{\underline{\underline{\mathcal{S}}}_R} \right)^{1-\phi+\sigma} = \frac{\iota_H}{\iota_L} \quad (28)$$

Using first order conditions for \mathcal{S}_H , $\tilde{\mathcal{S}}_H$ and $\underline{\mathcal{S}}_H$, one can derive

$$\iota_H \underline{\mathcal{S}}_H^{1-\sigma} \overline{\mathcal{S}}_R^{\phi-\sigma} = \iota_L \tilde{\mathcal{S}}_H^{1-\sigma} \underline{\mathcal{S}}_R^{\phi-\sigma} \quad (29)$$

Combining (27), (28) and (29), following can be derived

$$\frac{\tilde{\mathcal{S}}_H}{\underline{\mathcal{S}}_H} = \left(\frac{\iota_H}{\iota_L} \right)^{\frac{1}{1-\phi}} \quad (30)$$

Using $\tilde{\mathcal{S}}_H + \underline{\mathcal{S}}_H = 2\mathcal{S}_H$ and (30), $\tilde{\mathcal{S}}_H$ and $\underline{\mathcal{S}}_H$ can be written as functions of \mathcal{S}_H . Plugging these expressions back into the objection function and taking first order conditions for $\overline{\overline{\mathcal{S}}}_R$, $\overline{\mathcal{S}}_R$, $\underline{\mathcal{S}}_R$ and $\underline{\underline{\mathcal{S}}}_R$ again yields

$$\begin{aligned} \frac{\overline{\overline{\mathcal{S}}}_R}{\overline{\mathcal{S}}_R} &= \left(\frac{(\iota_H/\iota_L)^{\frac{1}{1-\phi}}}{1 + (\iota_H/\iota_L)^{\frac{1}{1-\phi}}} \right)^{\frac{1}{1-\phi+\sigma}} \\ \frac{\underline{\underline{\mathcal{S}}}_R}{\underline{\mathcal{S}}_R} &= \left(\frac{1}{1 + (\iota_H/\iota_L)^{\frac{1}{1-\phi}}} \right)^{\frac{1}{1-\phi+\sigma}} \end{aligned} \quad (31)$$

Using equations in (31), $\overline{\overline{\mathcal{S}}}_R$ and $\underline{\underline{\mathcal{S}}}_R$ can be written down as functions of $\overline{\mathcal{S}}_R$ and $\underline{\mathcal{S}}_R$ respectively. Plugging these expressions back again to the objective function and rearranging gives the objective function form in (25). \square

Lemma 3 suggests, the problem of a firm with two branches can be thought of as the problem of a firm with single branch who faces a higher probability (π_H) of high productivity

state and a lower probability (π_L) of low productivity state³².

Lemma 4. *For a given w_S , additional value from branching can be written as*

$$\Pi^B(\zeta) - \Pi(\zeta) = \zeta^{\frac{1}{1-\phi}} \left(\iota_{c'}^{\frac{1-\phi+\sigma}{1-\phi}} - \iota_c^{\frac{1-\phi+\sigma}{1-\phi}} \right) \mathcal{F}(w_S, \kappa) - \Upsilon \quad (32)$$

where

$$\iota_{c'} = \left((\pi_H \iota_H^{\frac{1}{1-\phi+\sigma}} + \pi_L \iota_L^{\frac{1}{1-\phi+\sigma}})^{\frac{1-\phi+\sigma}{1-\phi}} \right)$$

and $\mathcal{F}(w_S, \kappa)$ is decreasing in both arguments.

Proof. See Appendix 8.2 □

Lemma 37 suggests benefits from extending to two branches strictly increases with managerial ability ζ . Given the cost of extending Υ is fixed, Corollary 1 is immediate.

Corollary 1. *There exists a threshold skill level $b \in [u, \infty)$ s.t. all managers with the skill level above b would switch to two branches while those below would stick to a single branch.*

Using Corollary 1, we can reduce the equilibrium definition into prices, allocations, two threshold levels for occupational decisions l, u and one threshold for branching decision $b \geq u$. We are not particularly interested in how the availability of branching interacts with occupational decisions. Thus, we will focus on a partial equilibrium where l, u are fixed and analyze how changes in κ affect the wage structure under branching opportunities vs an economy with no branching. Lastly, we will focus on interior equilibria, i.e. values of Υ where $b > u$.

Proposition 4. *In an economy with branching opportunities, compared to an economy without branching opportunities, in equilibrium,*

- (1) *The fraction of professionals rented is the same.*
- (2) *The wages of laborers are the same while the wages of professionals are higher.*
- (3) *The dispersion of wages of professionals is higher.*
- (4) *The distribution of manager's (ex-ante) income has a smaller lower bound and a fatter tail.*

Proof. See Appendix 8.2 □

³²Realize that $\pi_H + \pi_L = 1$ is not necessarily satisfied.

Proposition 7 suggests, whether firms in a given economy consist of more or fewer branches doesn't have any implications on the size of the PBS sector. This is surprising given that the economy with more branches produces more efficiently and both professionals and managers benefit from the extra surplus. Lastly, earnings inequality is larger in the economy with more branching opportunities.

Proposition 5. *Holding l, u constant, as κ goes down,*

- (1) *Change in fraction of rented professionals does not depend on branching opportunities.*
- (2) *Wages of professionals increases faster in an economy with branching opportunities.*

Proof. Part (1) comes from Proposition 7. Since the fraction of rented professionals is the same between two economies for all values of κ , the change would also be identical.

Part (2) follows from (41) as $|\partial w_S / \partial \kappa|$ is larger in an economy with branching opportunities since $\iota_{c'} > \iota_c$. □

8 Conclusion

The paper argues how three important changes in the U.S. economy were connected. First, we have argued how improvements in the protection of trade secrets have contributed to the growth of Professional Business Services. Second, we have argued the growth in this sector has contributed to the increased wage inequality in the U.S. Economy. The channels paper represents have important implications for both the policymakers and the the academics.

As of 2019, EU countries are working on a framework that will standardize trade secret protection law across countries while the United Kingdom is on the brink of leaving the union and its requirements on the IP framework. On the other hand, according to estimates provided by the U.S. Chamber of Commerce, there are vast differences across countries in terms of the quality of trade secret protection. This paper discusses how a uniform and well-defined trade secret protection law can simplify business collaboration and allow the outsourcing of high-skill in the economy. Increased outsourcing improves the allocation of talent in the economy, however, it can also decrease entrepreneurship and increase wage inequality. The lawmakers should take all of these potential implications into account while designing new legal frameworks.

A large body of literature in Economics is on understanding the causes of increasing inequality in the U.S. The trend has been discussed as concerning for the bottom of the

distribution, while those on the top are perceived to be benefiting from the change. Some of the changes over the past decades may have allowed high-skilled to reach larger geography or consumers or augment their skills using new technology. However, this paper argues, some of the wage increase may be due to the increased intensity of the work schedule for the high-skilled. It is not clear whether high-skilled are indeed better off compared to four decades ago.

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NAICS Code	Subsector	% Revenue from Businesses	% Revenue from Government	% Revenue from Households
5411	Legal Services	67	6	28
5412	Accounting, Tax Preparation, Bookkeeping, and Payroll Services	71	4	25
5413	Architectural, Engineering, and Related Services	69	28	4
5414	Specialized Design Services	80	1	19
5415	Computer Systems Design and Related Services	68	32	0
5416	Management, Scientific, and Technical Consulting Services	73	20	7
5417	Scientific Research and Development Services	64	32	4
5418	Advertising and Related Services	90	3	7
5419	Other Professional, Scientific, and Technical Services	46	8	46

Table 6: Percentage distribution of revenue sources for NAICS54 sector, according to 2017 Q2& Q3 data from Quarterly Survey of Services

Appendix

8.1 PBS Employment

NAICS (North American Industry Classification System) gives the most obvious classification on PBS employment, however, data on relevant subsectors in NAICS classification only goes back to 1990. Previous to that, industries are separated with respect to SIC (Standard Industry Classification). We use crosswalk tables provided by U.S. Census to match relevant NAICS subsectors with those which are classified under U.S. Census 1990 classification. For that, we first need to identify which subsectors we are interested in NAICS classification.

NAICS does not completely distinguish firms regarding whether they serve businesses or consumers. However, subsectors of Professional, scientific, and technical services (NAICS 54) is the most relevant population of firms. The subsectors are given in Table 8.1, with the percentage of their revenue coming from businesses, government and households.

Almost half of the revenue of 'Other Professional, Scientific, and Technical Services' revenue comes from households. In addition, more than half of the employment in there is classified as 'veterinary services' and 'photographic services'. Hence, we will drop the subsector from our analysis for both questions, therefore drop Business Services n.e.c. which corresponds to NAICS 5419 in U.S. Census classification. Rest of the subsectors can be matched to the U.S. Census 1990 3-digit codes that are presented in the data section of the

main text.

8.2 Proofs

Proposition 6. *Under assumptions that imply perfect sorting into occupations, for a continuous skill distribution F and for the production function given in (8), equilibrium exists and it is unique.*

Proof. Manager's problem is concave in all its arguments, therefore first order conditions are sufficient. Using the first order conditions of the managers, market clearing conditions and indifference conditions, the problem can be reduced to three equations and three unknowns l, u, w_S :

$$\left(\frac{\sigma}{w_S}\right)^{\frac{1}{1-\phi}} \iota_c^{\frac{1+\sigma-\phi}{1-\phi}} \left(\left[\frac{\phi-\sigma}{\sigma\kappa}\right]^{\frac{\phi-\sigma}{1-\phi}} + \left[\frac{\phi-\sigma}{\sigma\kappa}\right]^{\frac{1-\sigma}{1-\phi}} \right) f_M^S = f_S \quad (33)$$

$$w_S = u^{\frac{\phi}{1-\phi}} \left(\frac{1}{w_S}\right)^{\frac{\phi}{1-\phi}} \left(\frac{1}{\kappa}\right)^{\frac{\phi-\sigma}{1-\phi}} \iota_c^{\frac{1+\sigma-\phi}{1-\phi}} \mathcal{G}(\phi, \sigma) + u^{\frac{\psi}{1-\psi}} (1-\psi) \left(\frac{\psi}{lw_S}\right)^{\frac{\psi}{1-\psi}} \quad (34)$$

$$lF(l)^{1-\psi} = \frac{\psi(f_M^L)^{1-\psi}}{w_S} \quad (35)$$

where $f_M^S = \int_u^\infty z^{1/(1-\phi)} dF(z)$, $f_M^L = \int_u^\infty z^{1/(1-\psi)} dF(z)$, $f_S = \int_l^\infty z dF(z)$ and $\iota_c = \pi \iota_H^{\frac{1}{1+\sigma-\phi}} + (1-\pi) \iota_L^{\frac{1}{1+\sigma-\phi}}$. Moreover $\mathcal{G}(\phi, \sigma) > 0$.

Let $l^*(u, w_S)$ be the solution to equation 35. LHS of (35) continuously increases from 0 to ∞ when F is continuous, therefore it is easy to show $l^*(u, w_S)$ exists and it is unique. Moreover, $\partial l^*/\partial u$ and $\partial l^*/\partial w_S$ are strictly negative. Taking the implicit solution for the equation 35 as given, we can reduce the system into unknowns u and w_S and equations (33) and (34), keeping the partial derivatives of l^* in mind.

Let $w_S^*(u)$ be the solution to equation 33 and $u^*(w_S)$ be the solution to equation 35. It is trivial to show both exist and are unique. We will show $u^*(w_S)$ and $w_S^*(u)$ have a unique crossing. First, $\lim_{u \rightarrow 0} w_S^*(u) = \infty$, $\lim_{u \rightarrow \infty} w_S^*(u) = 0$ and $dw_S^*(u)/du < 0$. Second, $\lim_{w_S \rightarrow 0} u^*(w_S) = 0$, $\lim_{w_S \rightarrow \infty} u^*(w_S) = \infty$ and $du^*(w_S)/dw_S > 0$. Therefore, $u^*(w_S)$ and $w_S^*(u)$ must have a unique crossing.

Lastly, given the values for u^* , l^* and w_S^* , we can uniquely pin down rest of the equilibrium objects using

$$\begin{aligned}
w_L &= \psi \left(\frac{f_M^L}{F(l)} \right)^{1-\psi} \\
S_H &= \left(\frac{\zeta\sigma}{w_S} \right)^{\frac{1}{1-\phi}} \left(\frac{\phi-\sigma}{\sigma\kappa} \right)^{\frac{\phi-\sigma}{1-\phi}} \iota_c^{\frac{1-\phi+\sigma}{1-\phi}} \\
\overline{S}_R &= \left(\frac{\zeta\sigma}{w_S} \right)^{\frac{1}{1-\phi}} \left(\frac{\phi-\sigma}{\sigma\kappa} \right)^{\frac{1-\sigma}{1-\phi}} \iota_c^{\frac{\sigma}{1-\phi}} \iota_H^{\frac{1}{1-\phi+\sigma}} \\
\underline{S}_R &= \left(\frac{\zeta\sigma}{w_S} \right)^{\frac{1}{1-\phi}} \left(\frac{\phi-\sigma}{\sigma\kappa} \right)^{\frac{1-\sigma}{1-\phi}} \iota_c^{\frac{\sigma}{1-\phi}} \iota_L^{\frac{1}{1-\phi+\sigma}}
\end{aligned} \tag{36}$$

and the profit function given in (9). Therefore, a unique equilibrium exists.

Uniqueness - Alternative Proof Given a pair $\{l, u\}$, we can uniquely pin down w_L from market clearing for labor and w_S from the indifference condition between becoming a worker vs a professional. Given the prices, we can uniquely pin down the allocations as well. Proving there cannot exist two equilibria with a different $\{l, u\}$ pair would be sufficient to conclude equilibrium is unique. Suppose, for a contradiction, that there exists two different equilibria \mathbf{e}, \mathbf{e}' with occupational thresholds $\{l, u\}$ and $\{l', u'\}$. Without loss of generality, assume $l' > l$. Let $u' \leq u$. Then, there would be less professional hours and more managerial technologies available in \mathbf{e}' compared to \mathbf{e} . Thus, $w'_S > w_S$ would immediately follow. Size of w_L versus w'_L depends on how relative sizes of laborers and managers differ. First, assume $w'_L \leq w_L$. Then, agent with skill level l would strictly prefer being a professional to a laborer, contradicting $l' > l$. Second, assume instead $w'_L > w_L$. Then both inputs become more expensive for managers and agent with skill level u' would strictly prefer being a professional to a manager, contradicting $u' \leq u$. Thus it cannot be the case that $u' \leq u$.

Now suppose instead $u' > u$. In this economy, there would be more basic labor hours and less managers and $w'_L < w_L$ would immediately follow. Size of w_S versus w'_S depends on how relative sizes of laborers and managers differ. First, assume $w'_S \geq w_S$. Then, agent with skill level l would strictly prefer being a professional to a laborer, contradicting $l' > l$. Second, assume instead $w'_S < w_S$. Then, both inputs for the managers become cheaper and agent with skill level u would strictly prefer being a manager to a professional, contradicting $u' > u$. Thus it cannot be the case that $u' > u$ either. Thus, there cannot exist two equilibria \mathbf{e} and \mathbf{e}' where $l' > l$. Lastly, assume $l' = l$. Either $u' > u$ or $u' < u$ has to be the case for \mathbf{e} and \mathbf{e}' to be distinct. If the agent u is indifferent between being a professional and a manager in equilibrium \mathbf{e} , then agent u' would strictly prefer one over the other in equilibrium \mathbf{e}' . Therefore \mathbf{e}' cannot be an equilibrium. \square

Lemma 5. For a given w_S , additional value from branching can be written as

$$\Pi^B(\zeta) - \Pi(\zeta) = \zeta^{\frac{1}{1-\phi}} \left(\iota_{c'}^{\frac{1-\phi+\sigma}{1-\phi}} - \iota_c^{\frac{1-\phi+\sigma}{1-\phi}} \right) \mathcal{F}(w_S, \kappa) - \Upsilon \quad (37)$$

where

$$\iota_{c'} = \left((\boldsymbol{\pi}_H \iota_H^{\frac{1}{1-\phi+\sigma}} + \boldsymbol{\pi}_L \iota_L^{\frac{1}{1-\phi+\sigma}}) \right)^{\frac{1-\phi+\sigma}{1-\phi}}$$

and $\mathcal{F}(w_S, \kappa)$ is decreasing in both arguments.

Proof. Solution to (25) would yield

$$\begin{aligned} \mathcal{S}_H &= \left(\frac{\zeta \sigma}{w_S} \right)^{\frac{1}{1-\phi}} \left(\frac{\phi - \sigma}{\sigma \kappa} \right)^{\frac{\phi - \sigma}{1-\phi}} \iota_{c'}^{\frac{1-\phi+\sigma}{1-\phi}} \\ \bar{\mathcal{S}}_R &= \left(\frac{\zeta \sigma}{w_S} \right)^{\frac{1}{1-\phi}} \left(\frac{\phi - \sigma}{\sigma \kappa} \right)^{\frac{\phi - \sigma}{1-\phi}} \iota_{c'}^{\frac{\sigma}{1-\phi}} \iota_H^{\frac{1}{1-\phi+\sigma}} \\ \underline{\mathcal{S}}_R &= \left(\frac{\zeta \sigma}{w_S} \right)^{\frac{1}{1-\phi}} \left(\frac{\phi - \sigma}{\sigma \kappa} \right)^{\frac{\phi - \sigma}{1-\phi}} \iota_{c'}^{\frac{\sigma}{1-\phi}} \iota_L^{\frac{1}{1-\phi+\sigma}} \end{aligned} \quad (38)$$

together with the conditions that were plugged back in

$$\begin{aligned} \tilde{\mathcal{S}}_H &= \mathcal{S}_H \left(\frac{(\iota_H / \iota_L)^{\frac{1}{1-\phi}}}{1 + (\iota_H / \iota_L)^{\frac{1}{1-\phi}}} \right)^{\frac{1}{\phi - \sigma}} \\ \tilde{\mathcal{S}}_H &= \mathcal{S}_H \left(\frac{1}{1 + (\iota_H / \iota_L)^{\frac{1}{1-\phi}}} \right)^{\frac{1}{\phi - \sigma}} \\ \underline{\mathcal{S}}_R &= \underline{\mathcal{S}}_R \left(\frac{1}{1 + (\iota_H / \iota_L)^{\frac{1}{1-\phi}}} \right)^{\frac{1}{1-\phi+\sigma}} \\ \bar{\mathcal{S}}_R &= \bar{\mathcal{S}}_R \left(\frac{(\iota_H / \iota_L)^{\frac{1}{1-\phi}}}{1 + (\iota_H / \iota_L)^{\frac{1}{1-\phi}}} \right)^{\frac{1}{1-\phi+\sigma}} \end{aligned} \quad (39)$$

Plugging all these expressions back to the profit function would give

$$\Pi^B(\zeta) = \zeta^{\frac{1}{1-\phi}} \iota_{c'}^{\frac{1-\phi+\sigma}{1-\phi}} \left(\frac{1}{w_S} \right)^{\frac{\phi}{1-\phi}} \left(\frac{1}{\kappa} \right)^{\frac{\phi - \sigma}{1-\phi}} \mathcal{G}(\phi, \sigma) + \zeta^{\frac{1}{1-\psi}} \left(\frac{1}{w_L} \right)^{\frac{\psi}{1-\psi}} (1 - \psi) \quad (40)$$

Profit function for the firm with a single branch would have the exact same expression with (40), replacing $\iota_{c'}$ with ι_c . The result follows from subtracting two profit functions. \square

Proposition 7. *In an economy with branching opportunities, compared to an economy without branching opportunities, in equilibrium,*

- (1) *Fraction of professionals rented is the same.*
- (2) *Wages of laborers are the same while wages of professionals are higher.*
- (3) *Dispersion of wages of professionals is higher.*
- (4) *Distribution of manager's (ex-ante) income has a smaller lower bound and a fatter tail.*

Proof. Realized skilled labor demand for a manager with two branches can be written as

$$\mathcal{S} = \left\{ \begin{array}{l} \mathcal{S}_H + \bar{\mathcal{S}}_R, \\ \mathcal{S}_H + \frac{\mathcal{S}_R}{2} \left(\frac{1}{1+(\nu_H/\nu_L)^{\frac{1}{1-\phi}}} \right)^{\frac{1}{1-\phi+\sigma}} + \frac{\bar{\mathcal{S}}_R}{2} \left(\frac{(\nu_H/\nu_L)^{\frac{1}{1-\phi}}}{1+(\nu_H/\nu_L)^{\frac{1}{1-\phi}}} \right)^{\frac{1}{1-\phi+\sigma}}, \\ \mathcal{S}_H + \underline{\mathcal{S}}_R, \end{array} \right. \begin{array}{l} \text{w.p. } \pi_H^2 \\ \text{w.p. } 2\pi_H\pi_L \\ \text{w.p. } \pi_L^2 \end{array}$$

This also implies the expected skilled labor demand for a manager is

$$E[\mathcal{S}] = \mathcal{S}_H + \pi_L \underline{\mathcal{S}}_R + \pi_H \bar{\mathcal{S}}_R$$

It can be seen that ratio of rented vs hired professionals does not depend on the probabilistic structure of the production function, which gives us part 1 of the Proposition. Market clearing condition for skilled labor would be

$$\int_u^b \left(\mathcal{S}_{H,i} + \pi_H \bar{\mathcal{S}}_{R,i} + \pi_L \underline{\mathcal{S}}_{R,i} \right) dF(i) + \int_b^\infty \left(\mathcal{S}_{H,i} + \pi_H \bar{\mathcal{S}}_{R,i} + \pi_L \underline{\mathcal{S}}_{R,i} \right) dF(i) = \int_l^u i dF(i)$$

which would give

$$w_S = \sigma \left[\left(\frac{\phi - \sigma}{\sigma \kappa} \right)^{\frac{\phi - \sigma}{1 - \phi}} + \left(\frac{\phi - \sigma}{\sigma \kappa} \right)^{\frac{1 - \sigma}{1 - \phi}} \right]^{1 - \phi} \left[\frac{\nu_c \int_b^\infty \zeta^{\frac{1}{1 - \phi}} dF(\zeta) + \nu_c \int_u^b \zeta^{\frac{1}{1 - \phi}} dF(\zeta)}{\int_l^u \zeta dF(\zeta)} \right] \quad (41)$$

Wages of laborers would still be given by

$$w_L = \psi \left(\frac{f_M^L}{F(l)} \right)^{1 - \psi}$$

since laborer demand is not affected by branching decision. This completes part 2 and

part 3 of the proposition as $\iota_{c'} > \iota_c$. Lastly, b would be pinned down by

$$b^{\frac{1}{1-\phi}} \left(\iota_{c'}^{\frac{1-\phi+\sigma}{1-\phi}} - \iota_c^{\frac{1-\phi+\sigma}{1-\phi}} \right) \mathcal{F}(w_S, \kappa) = \Upsilon \quad (42)$$

Given the wages of professionals are higher in the economy with branching option, those firms who are not branched make less profits in equilibrium. Moreover benefits from branching increases without bounds with ζ , therefore managers at the top of the skill distribution would make more profits in equilibrium. For those firms who are at the branching margin, whether they are better off or worse off depends on the magnitudes of the wage effect and branching effect.

□