Compatibility Strategies of Asymmetric Digital Platforms

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February 20 2019

Keywords: compatibility strategy, digital platform competition, asymmetric platforms, asymmetric application products

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He Liu and Han Qiao gratefully acknowledge the generous financial support from University of Chinese Academy of Sciences and General Program of National Natural Science Foundation of China (71872171).

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Abstract

We examine the profit-maximizing compatibility strategies in a duopoly setting of asymmetric platforms (a superior and an inferior platform) and asymmetric application products (a superior and an inferior product) by developing and analyzing a two stage game-theoretic model. We investigate two questions: (i) what are the profit-maximizing compatibility choices of each of the two firms (i.e., non-compatibility, one-way compatibility, or two-way compatibility); and (ii) how much to invest in adding intrinsic value to the compatible system (i.e., incremental compatibility investment). We find that the profit-maximizing compatibility choice depends on the adoption costs of a platform and/or an application product and on the intrinsic value difference between the two platforms and/or products. The larger the adoption costs, the greater the incentive for digital platforms to have compatible application products, as it increases profitability. We also show that the incremental compatibility investment is influenced by the intrinsic value difference between the digital platforms and/or application products. The larger that intrinsic value difference is, the larger should be the incremental compatibility investment when the superior product is made compatible and is being customized to the superior platform. This study contributes to the literature by: (i) filling a theoretical research void on compatibility strategies of revenue-generating asymmetric platforms with revenue-generating asymmetric application products; and (ii) showing the link between the size of the incremental compatibility investment, which is an endogenous firm decision variable in our model, and consumers' utility.

Keywords: compatibility strategy, digital platform competition, asymmetric platforms, asymmetric application products

1. Introduction

There has been a surge of coopetition[1] between digital platform firms (Adner et al. 2019); that is, an increasing number of digital platform firms compete and cooperate at the same time. Since digital platforms and digital application products are complementary to each consumer, many high-tech firms produce digital platforms and digital application products and sell them as a bundle. Asymmetry among digital platform firms is common: some firms have superior platforms, while others may have superior application products that run on digital platforms. One way of coopetition between asymmetric digital platforms is for one platform firm to make its products not only compatible with its own platform, but also compatible with competing platforms.

Apple and Microsoft are firms that produce both digital platforms and application products. However, the product-market strategies of Apple and Microsoft are distinct. The ecosystem Apple generates is quite exclusive because Apple's iOS operating system platform can only be installed on its own hardware. As well, third-party application products for Apple's hardware, such as MS Office for iPad, MS OneNote, MS OneDrive, and MS Edge, are available only through Apple's iTunes Store. Apple has the right to accept or reject a third-party application product for reasons that may include product quality or strategy concerns. Using a different product-market strategy, Microsoft (MS) cooperates with many third-party hardware manufacturers. It promotes the adoption of its Windows operating system platform by a range of hardware manufacturers. The Windows platform has been generally perceived to be inferior to Apple's iOS platform.[2] The benefits of the MS application suite Office, in which a number of applications such as a word processor and a spreadsheet are bundled, are highlighted by Gandal et al. (2018). While Apple has a similar application suite, iWork, it has generally been perceived to be inferior to MS Office. One key difference between the product-market strategies of these two companies is that Microsoft's Windows operating system platform is compatible with multiple hardware manufacturers, while Apple's superior iOS operating system platform is only compatible with Apple's hardware. Further, Microsoft's application product, MS Office, is compatible with any hardware that runs on the MS Windows operating system platform, whereas Apple's application product, iWork, is compatible only with Apple's hardware.[3]

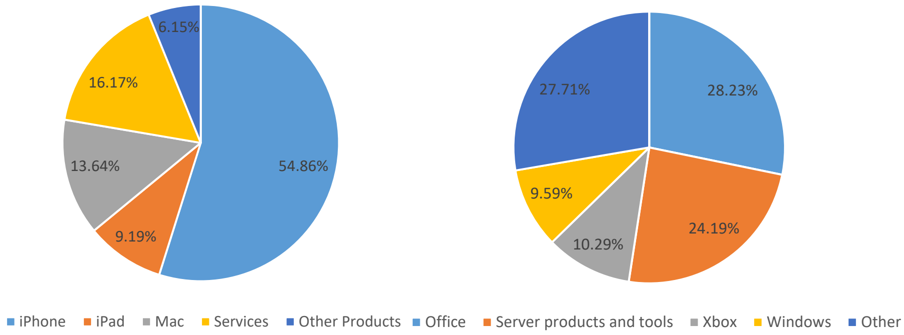
With different product-market strategies, Apple focuses on its own hardware and software, while Microsoft has mainly focused on its own software. According to Apple's fiscal year 2017 Q4

[1] See Brandenburger and Nalebuff (1996) for a primer of coopetition.

[2] iOS has been perceived as superior to Microsoft's Windows platform due in part to the GUI and the robustness of iOS.

[3] iTunes and iCloud are exceptions to the compatibility of Apple's application product with non-Apple hardware. The development of cloud computing enables subscribers to access Apple's applications on the cloud.

financial report (see Figure 1(a)), the revenue from iPhone, iPad, and Mac accounts for 54.86%, 9.19%, and 13.64% of the total revenue, respectively, which shows that revenue from the sale and usage of Apple's bundled hardware and software combination account for the vast majority of Apple's revenue. In contrast, the percentage of revenue from software is bigger than from hardware for Microsoft. According to Microsoft's 2017 annual report (see Figure 1(b)), the revenue from Microsoft Office system, Server products, and Windows PC operating system account for 28.23%, 24.19%, and 9.59% of total revenue, respectively. It is worth noting that the revenue from Microsoft Office system was around three times more than the revenue from Windows PC operating system platform, which indicates that the Office system is the largest revenue source of Microsoft Corporation.



(a) Apple: revenue breakdown by product

(b) Microsoft: revenue breakdown by product

Figure 1 Revenue breakdown by product of Apple and Microsoft

Although the main business of the two companies is different, Apple and Microsoft compete with each other in many areas. In Microsoft's 2017 annual report, the firm points out that Apple is Microsoft's main competitor in the field of operating systems, hardware devices, and games. Specifically, the Windows PC operating system faces competition from Apple's iOS system, Microsoft's hardware devices (Surface, PC accessories, and other intelligent devices, such as Surface Hub and HoloLens) face competition from Apple's iPad and Mac, and Microsoft's Xbox Live also faces competition from Apple TV.[4]

Despite the fierce competition between Apple and Microsoft, these two companies also cooperate with each other in an active way; that is, they compete and cooperate at the same time - coopetition (Brandenburger and Nalebuff 1996). The launch of Office system on Apple's devices is an example of coopetition between these two companies. While Microsoft Phone and Surface compete with

[4] Data Source: Annual Report 2017 of Microsoft: https://www.microsoft.com/investor/reports/ar17/index.html#

iPhone and iPad fiercely, Office for iPhone and Office for iPad were launched in June 2013 and March 2014, respectively. For these two products, users with a Microsoft account can use Office for free as long as the screen of Apple's device is not larger than 10.1 inches. In this way, there is no need for users of iPhone, iPad Air (9.7-inch screen size), and iPad mini (7.9-inch screen size) to pay subscription fees for Office 365. However, for users of iPad Pro with screen sizes of 10.5 inches and 12.9 inches, they must pay subscription fees to Microsoft for using Office. The payment is made through Apple's App Store and Apple receives 30% of the subscription fee in the process. Apple's iWork is not being made available on Windows operating system platform.

In this example, we note that Microsoft's superior application product, Office, is made compatible on Apple's hardware, which runs on its superior operating system platform, iOS. However, Apple's application software, iWork, cannot be installed on hardware that runs on MS Windows, which is generally perceived to be inferior to iOS.

In the above example there are two asymmetric operating system platforms (the superior Apple iOS platform and the inferior MS Windows platform) and two asymmetric application products (the superior MS Office and the inferior Apple iWork). We observe that the superior application product, MS Office, is compatible with the superior operating system platform, iOS, while the inferior application product, iWork, is not compatible with the inferior operating system, MS Windows. Other examples are that Microsoft Outlook, OneNote, OneDrive, Edge, Teams, etc. are also available on Apple's devices. These examples raise the question of whether these are profit-maximizing compatibility strategies for Apple and Microsoft. More generally, what is the desired compatibility strategy in the presence of revenue-generating asymmetric platforms with asymmetric applications? This research sheds light on the compatibility choice and compatibility investment of asymmetric platforms with asymmetric application products. We develop and analyze a two stage game-theoretic model to address these issues.

The rest of this paper is organized as follows. We present a literature review in Section 2. In Section 3, we develop and analyze a two stage game-theoretic model and derive the equilibrium results. In Section 4, we compare different scenarios in details. Further, we develop a simulation in order to enable a comparison of profits under different scenarios. Finally, in Section 5, we summarize our research.

2. Literature Review

Our theoretical model draws on the "mix and match" compatibility literature (Economides 1989, Matutes and Regibeau 1992, Katz and Shapiro 1985, Kim and Choi 2015). The terminology "mix and match" means that consumers can mix the components from different manufacturers to suit their personal tastes. In this literature, multiproduct firms sell systems consisting of complementary components that cannot be used separately, but can be purchased separately (Matutes and

Regibeau 1992, Einhorn 1992). With compatibility, cutting the price of a component will lead to an increase in sales of systems using that component sold by each firm (including the rival firm). In this way, compatibility weakens each firm's incentives to cut prices because the price cut of any firm will partially benefit its rival (Einhorn 1992).

Much of the received literature examines the compatibility decisions in the case of symmetric firms. Only a few papers focus on the case when competing firms are asymmetric. Einhorn (1992) studies "mix and match" compatibility in vertically differentiated markets, where compatibility increases the degree of product (quality) differentiation and hence weakens the competition. Farrell et al. (1998) show that incompatibility will be attractive to firms when there are more than two firms with heterogeneous products. Choi (1996) examines the relationship between compatibility decisions and R&D incentives in a mix-and-match model. Denicolò (2000) studies an asymmetric case where one generalist firm offering both components of a system competes against two specialists each supplying one component only. Hahn and Kim (2012) examine how the firms' incentive for compatibility and its welfare effect are affected by the presence of asymmetry in system markets. Mantovani and Ruiz-Aliseda (2016) analyze the dark side of collaborating with complementors for innovation ecosystems. In all of the above cited papers, the compatibility decision is endogenous, while the investment in compatibility is exogenous. The model developed in this paper considers both the compatibility decisions and the compatibility investment decisions to be endogenous. The compatibility cost may include additional research and development expenditure (Matutes and Regibeau 1988), the costs of negotiating to select a standard, the costs of introducing a new, compatible product, and more (Katz and Shapiro 1985). These costs influence the quality of product compatibility and hence users' utility. There is a trade-off that must be considered by firms that face the compatibility decision: While compatibility enhances product differentiation, which might increase profits, compatibility investment adversely affects profits. Hence, it is necessary for system providers to optimize the size of their compatibility investment.

Our paper is also related to platform markets (Rochet and Tirole 2003, 2006, Caillaud and Jullien 2003, Athey et al. 2016, Bryan and Gans 2018, Belleflamme and Peitz 2018, Correia-da Silva et al. 2018). Much of the literature in this area studies competition between symmetric platform firms (Armstrong 2006, Armstrong and Wright 2007). Yet the research focusing on asymmetric platform competition is scant. Halaburda and Yehezkel (2018) examine the competition between focal and non-focal platforms that differ in quality, and show that the ability of the high-quality but non-focal platform to win the market is affected by the initial degree of focality. Casadesus-Masanell and Llanes (2015) investigate an open source software provider who competes with a for-profit provider of proprietary software. Casadesus-Masanell and Zhu (2010) study platform competition between an entirely ad-sponsored firm and a firm that is both subscription-based and ad-sponsored.

Gabszewicz and Wauthy (2014) investigate the competition between platforms with different network sizes. Some of the platform literature centers on compatibility: Boudreau (2010) investigates alternative opening strategies for technology platforms to complementors, namely granting greater access to the platform or giving up control over the platform. He finds that granting access fosters much higher levels of innovation by complementors than does giving up control over the platform. Doganoglu and Wright (2006) investigate the relationship between multi-homing[5] and compatibility and find that multi-homing makes compatibility less attractive to firms, but can increase the social desirability of compatibility. Casadesus-Masanell and Ruiz-Aliseda (2009) show that incompatibility is preferred by large platform firms because it can lead to market dominance and high profits. Maruyama and Zennyo (2013) find compatibility decision depends upon the stage of the product life cycle. Once many customers already own hardware devices and the sale of content becomes the major profit center, the competing firms have incentives to make content compatible. Adner et al. (2019) investigate two asymmetric competing platforms which provide users with different standalone utilities. In their paper, both platforms generate profits through hardware sales and royalties from third-party content providers. Among their findings, is the observation that incentives for platforms to establish one-way compatibility come from the difference in their profit foci, i.e., difference in profit from hardware sales and royalties. Our study builds on and complements Adner et al. (2019) by examining, within a game-theoretic duopoly setting, alternative compatibility strategies of two firms that produce revenue-generating asymmetric platforms and revenue-generating application products that are sold to consumers as a bundle. In evaluating the desirability of making its product available on a rival platform, each of the two firms in our setting needs to consider the various tradeoffs associated with compatibility, including: (i) the impact on the revenue generated from sales of its platform and product, (ii) the royalties a platform may realize (or pay) from the sales of a competing product on a competing platform, and (iii) the compatibility investment associated with making its product available on a competing platform. In making their purchase decision consumers consider the intrinsic value of the systems (platforms and application products) that are offered on the market, the price of the system and the adoption costs associate with the system.

3. The Model

Consider two digital platform companies in a duopoly market, firm 1 and firm 2. Firm $i (i = 1, 2)$ has its own platform X_i and application product Y_i sold through its platform. Platform X and

[5] Multi-homing refers to a scenario in which a fraction of the users adopt several platforms (Rochet and Tirole 2003). For example, some users install both the Internet Explorer and the Chrome browser on their PC. In contrast, when users adopt only one platform, we refer to this scenario a single-homing.

product Y sold through each platform are complementary to each consumer: That is, platform X and product Y are sold together as a system. The price of platform X_i is p_{X_i} and the price of product Y_i is p_{Y_i}. Consistent with Chao and Derdenger (2013) and Papanastasiou et al. (2017), we assume that each platform or product provides an intrinsic value for consumers. Platform X_1 and product Y_2 offer a superior intrinsic value to the consumers relative to platform X_2 and product Y_1.

In the non-compatible case, each firm sells its product exclusively through its own platform, and there are two systems in the market: X_1Y_1 and X_2Y_2. Their prices are $p_{X_1Y_1} = p_{X_1} + p_{Y_1};\quad p_{X_2Y_2} = p_{X_2} + p_{Y_2}$, respectively.

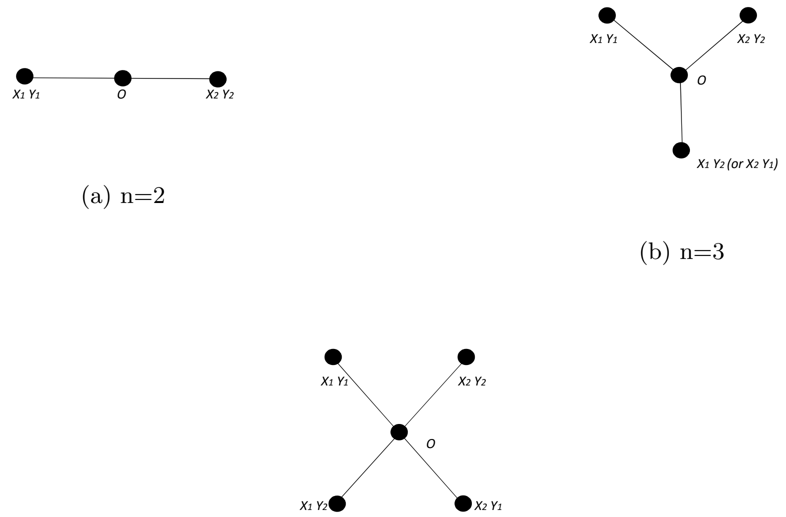
We analyze three compatibility scenarios: one-way compatibility with the superior product sold on both platforms, one-way compatibility with the inferior product sold on both platforms, and two-way compatibility. For one-way compatibility, we first investigate the scenario in which the superior product Y_2 is sold on both platforms and the inferior product Y_1 is only sold on platform 1. In this scenario, there are three systems in the market: X_1Y_1, X_2Y_2, X_1Y_2. Their prices are $p_{X_1Y_1} = p_{X_1} + p_{Y_1};\quad p_{X_2Y_2} = p_{X_2} + p_{Y_2};\quad p_{X_1Y_2} = p_{X_1} + p_{Y_2}$, respectively. We proceed to investigate the scenario in which the inferior product Y_1 is sold on both platforms and the superior product Y_2 is only sold on the inferior platform X_2. In this scenario, there are three systems in the market: X_1Y_1, X_2Y_2, X_2Y_1. Their prices are $p_{X_1Y_1} = p_{X_1} + p_{Y_1};\quad p_{X_2Y_2} = p_{X_2} + p_{Y_2};\quad p_{X_2Y_1} = p_{X_2} + p_{Y_1}$, respectively.

In the case of two-way compatibility, products Y_1, Y_2 are sold by both platforms. Therefore, there are four systems in the market: $X_1Y_1, X_2Y_2, X_1Y_2, X_2Y_1$, and their prices are $p_{X_1Y_1} = p_{X_1} + p_{Y_1};\quad p_{X_2Y_2} = p_{X_2} + p_{Y_2};\quad p_{X_1Y_2} = p_{X_1} + p_{Y_2};\quad p_{X_2Y_1} = p_{X_2} + p_{Y_1}$, respectively. We use superscripts "N, OS, OI, T" to denote the results for the Non-compatible case, the One-way compatible case with the Superior product sold on both platforms, the One-way compatible case with the Inferior product sold on both platforms, and the Two-way compatible case in which both products are sold on both platforms, respectively.

We formulate a Spokes model (Chen and Riordan 2007) to analyze the compatibility decision. The non-compatibility, one-way compatibility, and two-way compatibility correspond to $n = 2$, $n = 3$, and $n = 4$, respectively. In the Spokes model, the center point is denoted as O, and the distance between each end point and the center point is $\frac{1}{2}$. The unit transportation cost[6] is denoted by $t \geq 0$. Suppose that the total number of consumers in the market is 1 and they are uniformly distributed in the spokes. We also assume that each consumer will purchase a system consisting of a product and a platform from the systems that are offered on the market. That is, we assume

[6] Transportation cost in the context of a digital platform denotes adoption cost.

that the market is fully covered.[7] $d_{X_{i_1}Y_{j_1}}^{X_{i_2}Y_{j_2}}(i_1, i_2, j_1, j_2 = 1, 2, (i_1, j_1) \neq (i_2, j_2))$ represents the distance[8] between consumers and $X_{i_1}Y_{j_1}$ when they compare systems $X_{i_1}Y_{j_1}$ and $X_{i_2}Y_{j_2}$. We use $Q_{X_{i_1}Y_{j_1}}$ to denote the quantity of system $X_{i_1}Y_{j_1}$. In deciding which system to purchase a consumer will consider the adoption costs (which in the context of the model are referred to as transportation costs), the intrinsic value of the system, and the price of the system. The consumer will purchase the system that yields the highest net utility.



(a) n=2

(b) n=3

(c) n=4

Figure 2 The Spokes Model

The intrinsic value of system $X_iY_j(i, j = 1, 2)$ is denoted by $v_{X_iY_j}$.[9] Recalling that firm 1 has the superior platform X_1 and the inferior product Y_1 , and firm 2 has the inferior platform X_2 and the superior product Y_2, we assume $v_{X_1Y_1} = v_{X_2Y_2}$. This indicates that the intrinsic value of the system consisting of the superior platform X_1 and the inferior product Y_1 is the same as that of the system consisting of the inferior platform X_2 and the superior product Y_2. Recall that platform X_1 and product Y_2 offer greater intrinsic value to the consumer than platform X_2 and product Y_1. Here we denote the intrinsic value difference between X_1, the superior platform, and X_2, the inferior platform, by v_d, which is assumed to be positive. We further assume that $v_d>0$ also denotes the

[7] This assumption is commonly used in platform literature (e.g., Armstrong 2006, Adner et al. 2019).

[8] Distance in the context of a digital platform is the magnitude of the adoption cost.

[9] The intrinsic value of system X_iY_j is also known as the gross utility of consumers purchasing X_iY_j. The parameter $v_{X_iY_j}$ is assumed to be large enough so that the market is fully covered.

intrinsic value difference between Y_2 , the superior product, and Y_1, the inferior product. Under a compatible scenario where system X_iY_j exists in the market, we assume that firm j first invests I_0 to make its product Y_j function normally on platform X_i. This basic compatibility investment I_0 is the minimum fixed cost needed to make firm j's product Y_j compatible with platform X_i. In addition to basic compatibility investment, firm j may also have an incentive to invest additional sums beyond what is needed for basic compatibility in order to further customize its product Y_j on platform X_i and thereby enhance the intrinsic value of system X_iY_j. The incremental compatibility investment, $I_{Y_j}^{X_i}$, is a decision variable of firm j. The larger the incremental compatibility investment $I_{Y_j}^{X_i}$, the greater the intrinsic value consumers derive from purchasing system X_iY_j. For simplicity, we assume that this relationship between the intrinsic value of system X_iY_j and the incremental compatibility investment is linear. The difference in the utility of consumers who purchase X_1Y_2 and those who purchase X_2Y_2, lies in the intrinsic value difference v_d from the superior platform and from firm 2's incremental compatibility investment in product Y_2. Therefore, the intrinsic value of system X_1Y_2, which is denoted by $v_{X_1Y_2}$, compared to the intrinsic value of the system X_2Y_2, which is denoted by $v_{X_2Y_2}$, is: $v_{X_1Y_2} = v_{X_2Y_2} + v_d + I_{Y_2}^{X_1}$. The intrinsic value of system X_2Y_1 relative to system X_1Y_1 is $v_{X_2Y_1} = v_{X_1Y_1} - v_d + I_{Y_1}^{X_2}$.

3.1. Scenario 1: Competition in the non-compatible case (N Scenario)

In the non-compatible case, there are two systems in the market, X_1Y_1, X_2Y_2. The net utility of consumers purchasing X_1Y_1 is $U_{X_1Y_1} = v_{X_1Y_1} - p_{X_1Y_1} - td_{X_1Y_1}^{X_2Y_2}$, and the net utility of purchasing X_2Y_2 is $U_{X_2Y_2} = v_{X_2Y_2} - p_{X_2Y_2} - t(1 - d_{X_1Y_1}^{X_2Y_2})$, since we assumed that the market is fully covered. Let $U_{X_1Y_1} = U_{X_2Y_2}$ and notice that $v_{X_1Y_1} = v_{X_2Y_2}$, we can derive the location of the consumers who are indifferent between purchasing X_1Y_1 and X_2Y_2, since they derive the same net utility.

$$d_{X_1Y_1}^{X_2Y_2} = \frac{t + p_{X_2Y_2} - p_{X_1Y_1}}{2t}, \tag{1}$$

We use $Q_{X_1Y_1}, Q_{X_2Y_2}$ to denote the quantities of system X_1Y_1 and X_2Y_2, respectively. Comparing with the indifferent consumers, consumers who are located closer to X_1Y_1 choose X_1Y_1, while consumers located closer to X_2Y_2 choose X_2Y_2. Consistent with the Spokes model and the assumptions that consumers are uniformly distributed and the market is fully covered, we obtain $Q_{X_1Y_1} = d_{X_1Y_1}^{X_2Y_2}$.

Thus, the quantities are

$$Q_{X_1Y_1} = d_{X_1Y_1}^{X_2Y_2} = \frac{t + p_{X_2Y_2} - p_{X_1Y_1}}{2t}, \tag{2}$$

$$Q_{X_2Y_2} = 1 - Q_{X_1Y_1} = \frac{t + p_{X_1Y_1} - p_{X_2Y_2}}{2t}. \tag{3}$$

Consistent with the platform literature (see for example, Armstrong 2006, Adner et al. 2019, Mantovani and Ruiz-Aliseda 2016), we assume that the marginal costs of the platforms and products are zero and the fixed cost of each of the two firms is $C > 0$. The profit of firm 1 is $\pi_1^N = p_{X_1Y_1}Q_{X_1Y_1} - C$, and the profit of firm 2 is $\pi_2^N = p_{X_2Y_2}Q_{X_2Y_2} - C$. Maximizing the profit with respect to $p_{X_iY_i}$ yields the following first-order conditions

$$\begin{cases} \dfrac{\partial \pi_1^N}{\partial p_{X_1Y_1}} = \dfrac{t + p_{X_2Y_2} - 2p_{X_1Y_1}}{2t} = 0, \\ \dfrac{\partial \pi_2^N}{\partial p_{X_2Y_2}} = \dfrac{t + p_{X_1Y_1} - 2p_{X_2Y_2}}{2t} = 0. \end{cases} \tag{4}$$

Solving the first-order conditions, we obtain the profit-maximizing prices in the case of incompatible systems

$$p_{X_1Y_1}^N = p_{X_2Y_2}^N = t. \tag{5}$$

Substituting Eq. (5) into Eq. (2) and Eq. (3), we derive the quantities

$$Q_{X_1Y_1}^N = Q_{X_2Y_2}^N = \frac{1}{2}. \tag{6}$$

Substituting Eq. (6) and Eq. (5) into the profit function, we derive the profits of each firm under the non-compatible scenario denoted by N

$$\pi_1^N = \pi_2^N = \frac{t}{2} - C. \tag{7}$$

3.2. Scenario 2: One-way compatible case: the superior product sold on both platforms (OS Scenario)

In this scenario, the superior product Y_2 is sold by both platforms and the inferior product Y_1 is only sold by the superior platform X_1. Therefore the systems offered on the market are: X_1Y_1, X_2Y_2, X_1Y_2. The game consists of two stages. In the first stage, firm 2 decides on the incremental compatibility investment in product Y_2; in the second stage, firm 1 and firm 2 decide independently on the price of X_1, Y_1, X_2, Y_2, respectively, and consumers make their purchase decisions. The quantity is determined by the model based on the price and the incremental investment. We use backward induction to solve the game.

3.2.1. Stage 2 As stated, in the one-way compatible case there are three systems in the market, X_1Y_1, X_2Y_2, X_1Y_2. The intrinsic value of X_1Y_1 is $v_{X_1Y_1}$, the intrinsic value of X_2Y_2 is $v_{X_2Y_2}$, and the intrinsic value of X_1Y_2 is $v_{X_1Y_2} = v_{X_2Y_2} + v_d + I_{Y_2}^{X_1}$.

When consumers compare X_1Y_1 with X_2Y_2, the net utility of the consumer choosing X_1Y_1 is $v_{X_1Y_1} - p_{X_1Y_1} - td_{X_1Y_1}^{X_2Y_2}$, and the net utility of the consumer choosing X_2Y_2 is $v_{X_2Y_2} - p_{X_2Y_2} - t(1 - d_{X_1Y_1}^{X_2Y_2})$. Letting $v_{X_1Y_1} - p_{X_1Y_1} - td_{X_1Y_1}^{X_2Y_2} = v_{X_2Y_2} - p_{X_2Y_2} - t(1 - d_{X_1Y_1}^{X_2Y_2})$ we can derive the location of the indifferent consumers who are indifferent between purchasing X_1Y_1, X_2Y_2, since they derive the same net utility

$$d_{X_1Y_1}^{X_2Y_2} = \frac{t + v_{X_1Y_1} - v_{X_2Y_2} + p_{X_2Y_2} - p_{X_1Y_1}}{2t}. \tag{8}$$

Since there are three systems in the market, in addition to comparing X_1Y_1 with X_2Y_2, consumers who consider purchasing X_1Y_1 also compare it with X_1Y_2. When consumers make this comparison, the net utility for the consumer choosing X_1Y_1 is $v_{X_1Y_1} - p_{X_1Y_1} - td_{X_1Y_1}^{X_1Y_2}$, and the net utility for the consumer choosing X_1Y_2 is $v_{X_1Y_2} - p_{X_1Y_2} - t(1 - d_{X_1Y_1}^{X_1Y_2})$. By equating $v_{X_1Y_1} - p_{X_1Y_1} - td_{X_1Y_1}^{X_1Y_2} = v_{X_1Y_2} - p_{X_1Y_2} - t(1 - d_{X_1Y_1}^{X_1Y_2})$, we derive the location of the indifferent consumer choosing between X_1Y_1 and X_1Y_2

$$d_{X_1Y_1}^{X_1Y_2} = \frac{t + v_{X_1Y_1} - v_{X_1Y_2} + p_{X_1Y_2} - p_{X_1Y_1}}{2t}. \tag{9}$$

Consistent with the Spokes model, we have the following expressions for quantities of X_1Y_1

$$Q_{X_1Y_1} = \frac{2(d_{X_1Y_1}^{X_2Y_2} + d_{X_1Y_1}^{X_1Y_2})}{3*2} = \frac{2t + 2v_{X_1Y_1} - v_{X_1Y_2} - v_{X_2Y_2} - 2p_{X_1Y_1} + p_{X_1Y_2} + p_{X_2Y_2}}{6t}. \tag{10}$$

Similarly, we obtain the following quantities of systems X_1Y_2 and X_2Y_2

$$Q_{X_1Y_2} = \frac{2t + 2v_{X_1Y_2} - v_{X_1Y_1} - v_{X_2Y_2} - 2p_{X_1Y_2} + p_{X_1Y_1} + p_{X_2Y_2}}{6t}, \tag{11}$$

$$Q_{X_2Y_2} = \frac{2t + 2v_{X_2Y_2} - v_{X_1Y_1} - v_{X_1Y_2} - 2p_{X_2Y_2} + p_{X_1Y_1} + p_{X_1Y_2}}{6t}. \tag{12}$$

We assume that the royalty rate is a constant $b \in (0,1)$ multiplied by the quantity of the new system $Q_{X_1Y_2}$. In other words, firm 1 charges firm 2 $bQ_{X_1Y_2}$ for each unit product Y_2 sold through platform X_1.[10] We make this assumption for two reasons. First, introducing the superior product Y_2 on the superior platform X_1 may hurt the sales of its inferior product Y_1. As the quantity $Q_{X_1Y_2}$ of system X_1Y_2 increases, the quantity of firm 1's own system X_1Y_1 may decrease, which may hurt the profits of firm 1 from selling its system X_1Y_1. Therefore, to protect the sales of X_1Y_1, firm 1 has an incentive to increase the royalty rate paid by firm 2 to firm 1 for selling Y_2 on X_1.[11] Second, a larger $Q_{X_1Y_2}$ indicates that firm 2 relies more on firm 1's superior platform X_1 than on its own inferior platform X_2 to sell its superior product Y_2. With the royalty rate set at $bQ_{X_1Y_2}$ by firm 1, firm 2 needs to decide whether it wishes to list its product Y_2 on platform X_1. This formulation of

[10] For example, $b = 0.01$ implies that the royalty rate equals 1% of the amount of product Y_2 sold on platform X_1; that is, firm 2 pays firm 1 $0.01 * Q_{X_1Y_2}$: so if $Q_{X_1Y_2} = 0.4$, then for every unit Y_2 sold on platform X_1, firm 2 pays 0.004 to firm 1.

[11] Note: $Q_{X_1Y_2}$ is endogenously determined by the model.

the royalty rate is of course very costly for firm 2 and it can be viewed as an extreme measure by firm 1 to protect the sales of its inferior product Y_1.

The total amount that firm 1 charges firm 2 for firm 2's superior product Y_2's usage of firm 1's superior platform X_1 is $bQ^2_{X_1Y_2}$. In addition to the basic fixed compatibility investment I_0, we assume that the incremental compatibility cost of firm 2 to cover the incremental compatibility investment $I^{X_1}_{Y_2}$ is $(I^{X_1}_{Y_2})^2$.[12] Therefore, the profit of firm 1 is $\pi_1^{\mathrm{OS}} = p_{X_1Y_1}Q_{X_1Y_1} + p_{X_1}Q_{X_1Y_2} + bQ^2_{X_1Y_2} - C$, and the profit of firm 2 is $\pi_2^{\mathrm{OS}} = p_{X_2Y_2}Q_{X_2Y_2} + p_{Y_2}Q_{X_1Y_2} - bQ^2_{X_1Y_2} - I_0 - (I^{X_1}_{Y_2})^2 - C$. In order to optimize p_{X_i}, p_{Y_i} to maximize the profit, we assume $b < 3t$, which is a concave condition. Assuming that $b < 3t$ (which is a concavity condition), we maximize the profits of the two firms with respect to p_{X_i}, p_{Y_i}, and by solving the first-order conditions, we have the optimal prices

$$\begin{cases} p^{OS}_{X_1} = \dfrac{-2bI^{X_1}_{Y_2} + 3tI^{X_1}_{Y_2} - 2bt - 2bv_d + 3tv_d + 12t^2}{9t}, \\ p^{OS}_{Y_1} = \dfrac{2bI^{X_1}_{Y_2} - 3tI^{X_1}_{Y_2} + 2bt + 2bv_d - 3tv_d + 6t^2}{9t}, \\ p^{OS}_{X_2} = \dfrac{-2bI^{X_1}_{Y_2} - 3tI^{X_1}_{Y_2} - 2bt - 2bv_d - 3tv_d + 6t^2}{9t}, \\ p^{OS}_{Y_2} = \dfrac{2bI^{X_1}_{Y_2} + 3tI^{X_1}_{Y_2} + 2bt + 2bv_d + 3tv_d + 12t^2}{9t}. \end{cases} \tag{13}$$

The optimal quantities of systems are

$$\begin{cases} Q^{OS}_{X_1Y_1} = \dfrac{-I^{X_1}_{Y_2} + 8t - v_d}{18t}, \\ Q^{OS}_{X_1Y_2} = \dfrac{I^{X_1}_{Y_2} + t + v_d}{9t}, \\ Q^{OS}_{X_2Y_2} = \dfrac{-I^{X_1}_{Y_2} + 8t - v_d}{18t}. \end{cases} \tag{14}$$

3.2.2. Stage 1 Firm 2 maximizes π_2^{OS} by optimizing $I^{X_1}_{Y_2}$. Here we assume $81t^2 - 3t - b > 0$.[13]

The optimal incremental compatibility investment is obtained by solving the first-order conditions

$$I^{X_1}_{Y_2} = \frac{bt + bv_d + 3tv_d + 3t^2}{81t^2 - 3t - b}. \tag{15}$$

Substituting Eq. (15) into Eq. (13) and noting that $p_{X_1Y_1} = p_{X_1} + p_{Y_1}$; $p_{X_1Y_2} = p_{X_1} + p_{Y_2}$; $p_{X_2Y_2} = p_{X_2} + p_{Y_2}$, we derive the optimal prices of systems

$$\begin{cases} p^{OS}_{X_1Y_1} = 2t, \\ p^{OS}_{X_1Y_2} = 2t + \dfrac{54t^2(t + v_d)}{81t^2 - 3t - b}, \\ p^{OS}_{X_2Y_2} = 2t. \end{cases} \tag{16}$$

[12] Similar assumptions are commonly used in R&D literature (Mantovani and Ruiz-Aliseda 2016).

[13] This assumption guarantees the existence of the optimal solution.

Substituting Eq. (15) into Eq. (14), the equilibrium quantities are

$$\begin{cases} Q_{X_1Y_1}^{OS} = \dfrac{1}{2} - \dfrac{9t(t+v_d)}{2(81t^2-3t-b)}, \\ Q_{X_1Y_2}^{OS} = \dfrac{9t(t+v_d)}{81t^2-3t-b}, \\ Q_{X_2Y_2}^{OS} = \dfrac{1}{2} - \dfrac{9t(t+v_d)}{2(81t^2-3t-b)}. \end{cases} \tag{17}$$

Substituting Eq. (15), Eq. (16) and Eq. (17) into the profit function of each firm, we obtain the profit

$$\begin{cases} \pi_1^{OS} = t + \dfrac{81t^2(t+v_d)^2(3t-b)}{(81t^2-3t-b)^2} - C, \\ \pi_2^{OS} = t + \dfrac{(t+v_d)^2(b+3t)}{81t^2-3t-b} - I_0 - C. \end{cases} \tag{18}$$

We summarize the results under the one-way compatibility with the superior product sold on both platforms in the following proposition:

Proposition 1. *The equilibrium incremental compatibility investment by firm 2 in making its superior product Y_2 more valuable when it runs on the superior platform X_1, $I_{Y_2}^{X_1}$, is positive and increases in the intrinsic value difference between the superior platform/product and the inferior platform/product, v_d. As well, the price and quantity of X_1Y_2, $p_{X_1Y_2}^{OS}$ and $Q_{X_1Y_2}^{OS}$, increase in the intrinsic value difference, v_d. The profits of each firm, π_1^{OS} and π_2^{OS}, increase in the intrinsic value difference, v_d.*

Under one-way compatibility with the superior product Y_2 sold on both platforms, three systems exist in the market: X_1Y_1, X_2Y_2, X_1Y_2. Among them, system X_1Y_2, the superior platform with the superior product, offers the highest intrinsic value to consumers. X_1Y_1 and X_2Y_2 offer the same intrinsic value to consumers. The final intrinsic value difference, i.e., the system differentiation, between system X_1Y_2 and system X_1Y_1 or X_2Y_2 depends on the intrinsic value difference v_d, and the incremental investment by firm 2 to make its superior product Y_2 more valuable when it runs on the superior platform X_1.

When the intrinsic value difference between the superior platform X_1 and the inferior platform X_2 increases and/or the intrinsic value difference between the superior product Y_2 and the inferior product Y_1 increases, firm 2 has a greater incentive to increase its incremental investment to further customize and increase the intrinsic value of system X_1Y_2. By increasing the incremental investment to make Y_2 more valuable on the superior platform X_1, the intrinsic value of system X_1Y_2 is higher, which in turn increases differentiation among the three systems in the market. Since we assumed that the market is fully covered, the greater system differentiation and the higher intrinsic value of the system X_1Y_2 enable firm 1 to raise the price of X_1 and firm 2 to raise the price of Y_2, while the quantity of the system X_1Y_2 also increases.

3.3. Scenario 3: One-way compatible case: the inferior product sold on both platforms (OI Scenario)

In this scenario, the inferior product Y_1 is sold on both platforms and the superior product Y_2 is only sold on platform X_2, and the systems existing in the market are X_1Y_1, X_2Y_2, X_2Y_1. The game consists of two stages. In the first stage, firm 1 decides the incremental compatibility investment in product Y_1; in the second stage, firm 1 and firm 2 decide independently on the price of X_1, Y_1, X_2, Y_2, respectively. We use backward induction to solve the game.

We derive the equilibrium results for this scenario using a similar methodology to the one we used in Scenario 2.[14] The equilibrium incremental investment by firm 1 to make its inferior product Y_1 more valuable when it runs on the inferior platform X_2 is

$$I_{Y_1}^{X_2} = \frac{bt - bv_d - 3tv_d + 3t^2}{81t^2 - 3t - b} = \frac{(3t+b)(t-v_d)}{81t^2 - 3t - b}. \tag{19}$$

The optimal prices of the systems that are offered on the market are

$$\begin{cases} p_{X_1Y_1}^{OI} = 2t, \\ p_{X_2Y_1}^{OI} = 2t + \dfrac{54t^2(t-v_d)}{81t^2 - 3t - b}, \\ p_{X_2Y_2}^{OI} = 2t. \end{cases} \tag{20}$$

The equilibrium quantities are

$$\begin{cases} Q_{X_1Y_1}^{OI} = \dfrac{1}{2} - \dfrac{9t(t-v_d)}{2(81t^2 - 3t - b)}, \\ Q_{X_2Y_1}^{OI} = \dfrac{9t(t-v_d)}{81t^2 - 3t - b}, \\ Q_{X_2Y_2}^{OI} = \dfrac{1}{2} - \dfrac{9t(t-v_d)}{2(81t^2 - 3t - b)}. \end{cases} \tag{21}$$

Substituting Eq. (20), Eq. (21)and Eq. (19) into the profit function of each firm, we derive the optimal profit of each firm

$$\begin{cases} \pi_1^{OI} = t + \dfrac{(b+3t)(t-v_d)^2}{81t^2 - 3t - b} - I_0 - C, \\ \pi_2^{OI} = t + \dfrac{81t^2(t-v_d)^2(3t-b)}{(81t^2 - 3t - b)^2} - C. \end{cases} \tag{22}$$

We summarize the results in the following proposition. (See proof in the appendix.)

Proposition 2. *The equilibrium incremental compatibility investment by firm 1 in making its inferior product Y_1 more valuable on the inferior platform X_2, $I_{Y_1}^{X_2}$, decreases in v_d. As well, the price and quantity of system X_2Y_1, $p_{X_2Y_1}^{OI}$ and $Q_{X_2Y_1}^{OI}$, decrease in the intrinsic value v_d. The profits of each firm, π_1^{OI} and π_2^{OI}, decrease in v_d.*

[14] For a detailed calculation, please see the appendix. We assumed that $81t^2 - 3t - b > 0$ to guarantee the existence of the optimal solution for profits of each firm. We also assumed that $t > v_d$ to guarantee positive quantities.

Our findings suggest that under one-way compatibility with the inferior product sold on both platforms, the incremental investment by firm 1 in adding intrinsic value to its inferior product Y_1 when it runs on the inferior platform X_2, $I_{Y_1}^{X_2}$, decreases in the intrinsic value difference between the superior platform/product X_1/Y_2 and the inferior platform/product X_2/Y_1, v_d. The greater the intrinsic value difference v_d, the lower the incentive for firm 1 to increase its incremental investment in making its inferior product Y_1 more valuable on the inferior platform X_2. This is because the greater the intrinsic value difference, the less productive is the incremental investment in customizing Y_1 on platform X_2, as consumers are unlikely to substantially increase their purchases of the inferior system X_2Y_1. This in turn explains the analytical finding that the price and quantity of system X_2Y_1 decrease in v_d. As well, we find an inverse relationship between v_d and the profits of each firm.

3.4. Scenario 4: Two-way compatible case (T Scenario)

For two-way compatibility, we assume that products Y_1, Y_2 are sold by both platforms. Therefore, there are four types of systems in the market: $X_1Y_1, X_2Y_2, X_1Y_2, X_2Y_1$, and their prices are $p_{X_1Y_1} = p_{X_1} + p_{Y_1}; p_{X_2Y_2} = p_{X_2} + p_{Y_2}; p_{X_1Y_2} = p_{X_1} + p_{Y_2}; p_{X_2Y_1} = p_{X_2} + p_{Y_1}$, respectively. As before, the game consists of two stages. In the first stage, each firm decides on the incremental compatibility investment for their products; in the second stage, each firm decides independently on the prices of its platform and its product, and consumers make their purchase decisions given the prices. We use backward induction to solve the two stage game, and establish profit-maximizing prices, quantities, and incremental investments.

3.4.1. Stage 2 In the two-way compatible case, there are four systems in the market, X_1Y_1, X_2Y_2, X_1Y_2, X_2Y_1. The gross utility of a consumer who chooses X_1Y_1 is $v_{X_1Y_1}$, the gross utility of a consumer who chooses X_2Y_2 is $v_{X_2Y_2}$, the gross utility of choosing X_1Y_2 is $v_{X_1Y_2} = v_{X_2Y_2} + v_d + I_{Y_2}^{X_1}$, and the gross utility of choosing X_2Y_1 is $v_{X_2Y_1} = v_{X_1Y_1} - v_d + I_{Y_1}^{X_2}$.

To establish the location of consumers who are indifferent between choosing X_1Y_1 and X_2Y_2, we note that the net utility of choosing X_1Y_1 is $v_{X_1Y_1} - p_{X_1Y_1} - td_{X_1Y_1}^{X_2Y_2}$, and the net utility of choosing X_2Y_2 is $v_{X_2Y_2} - p_{X_2Y_2} - t(1 - d_{X_1Y_1}^{X_2Y_2})$. Equating the net utilities $v_{X_1Y_1} - p_{X_1Y_1} - td_{X_1Y_1}^{X_2Y_2} = v_{X_2Y_2} - p_{X_2Y_2} - t(1 - d_{X_1Y_1}^{X_2Y_2})$, and solving for $d_{X_1Y_1}^{X_2Y_2}$, we establish the location of consumers who are indifferent between choosing X_1Y_1 and X_2Y_2

$$d_{X_1Y_1}^{X_2Y_2} = \frac{t + v_{X_1Y_1} - v_{X_2Y_2} + p_{X_2Y_2} - p_{X_1Y_1}}{2t}. \tag{23}$$

To establish the location of consumers who are indifferent between choosing X_1Y_1 and X_1Y_2, we note that the net utility for the consumer choosing X_1Y_1 is $v_{X_1Y_1} - p_{X_1Y_1} - td_{X_1Y_1}^{X_1Y_2}$, and the net utility for the consumer choosing X_1Y_2 is $v_{X_1Y_2} - p_{X_1Y_2} - t(1 - d_{X_1Y_1}^{X_1Y_2})$. Equating the net utilities and solving for $d_{X_1Y_1}^{X_1Y_2}$, we have the location of indifferent consumers choosing between X_1Y_1 and X_1Y_2

$$d_{X_1Y_1}^{X_1Y_2} = \frac{t + v_{X_1Y_1} - v_{X_1Y_2} + p_{X_1Y_2} - p_{X_1Y_1}}{2t}. \tag{24}$$

Similarly, to establish the location of consumers who are indifferent between choosing X_1Y_1 and X_2Y_1, we note that the net utility for the consumer choosing X_1Y_1 is $v_{X_1Y_1} - p_{X_1Y_1} - td_{X_1Y_1}^{X_2Y_1}$, and the net utility for the consumer choosing X_2Y_1 is $v_{X_2Y_1} - p_{X_2Y_1} - t(1 - d_{X_1Y_1}^{X_2Y_1})$. Hence, we have the location of indifferent consumers choosing between X_1Y_1 and X_2Y_1

$$d_{X_1Y_1}^{X_2Y_1} = \frac{t + v_{X_1Y_1} - v_{X_2Y_1} + p_{X_2Y_1} - p_{X_1Y_1}}{2t}. \tag{25}$$

Consistent with the Spokes model, we derive the following expressions for quantities of the system X_1Y_1

$$\begin{aligned} Q_{X_1Y_1} &= \frac{2(d_{X_1Y_1}^{X_2Y_2} + d_{X_1Y_1}^{X_1Y_2} + d_{X_1Y_1}^{X_2Y_1})}{4*3} \\ &= \frac{3t + 3v_{X_1Y_1} - v_{X_1Y_2} - v_{X_2Y_2} - v_{X_2Y_1} - 3p_{X_1Y_1} + p_{X_1Y_2} + p_{X_2Y_1} + p_{X_2Y_2}}{12t}. \end{aligned} \tag{26}$$

Similarly, we obtain the following expressions for quantities of the systems X_1Y_2, X_2Y_1, X_2Y_2

$$Q_{X_1Y_2} = \frac{3t + 3v_{X_1Y_2} - v_{X_1Y_1} - v_{X_2Y_2} - v_{X_2Y_1} - 3p_{X_1Y_2} + p_{X_1Y_1} + p_{X_2Y_1} + p_{X_2Y_2}}{12t}, \tag{27}$$

$$Q_{X_2Y_1} = \frac{3t + 3v_{X_2Y_1} - v_{X_1Y_1} - v_{X_2Y_2} - v_{X_1Y_2} - 3p_{X_2Y_1} + p_{X_1Y_1} + p_{X_1Y_2} + p_{X_2Y_2}}{12t}, \tag{28}$$

$$Q_{X_2Y_2} = \frac{3t + 3v_{X_2Y_2} - v_{X_1Y_1} - v_{X_1Y_2} - v_{X_2Y_1} - 3p_{X_2Y_2} + p_{X_1Y_1} + p_{X_2Y_1} + p_{X_1Y_2}}{12t}. \tag{29}$$

As in the previous scenarios, we assume that the royalty rate is a constant $b \in (0,1)$ multiplied by the quantity of the new system. In other words, firm 1 charges firm 2 $bQ_{X_1Y_2}$ for each unit product Y_2 sold through platform X_1 and firm 2 charges firm 1 $bQ_{X_2Y_1}$ for each unit product Y_1 sold through platform X_2.[15] Hence, the royalty payment of firm 1 to firm 2 is $bQ_{X_2Y_1}^2$, and the royalty payment of firm 2 to firm 1 is $bQ_{X_1Y_2}^2$. In addition to the basic compatibility investment I_0, the compatibility costs of firm i to cover the incremental compatibility investment $I_{Y_i}^{X_j}$ is $(I_{Y_i}^{X_j})^2$. Therefore, the profit of firm 1 is $\pi_1 = p_{X_1Y_1}Q_{X_1Y_1} + p_{X_1}Q_{X_1Y_2} + p_{Y_1}Q_{X_2Y_1} + bQ_{X_1Y_2}^2 - bQ_{X_2Y_1}^2 - I_0 - (I_{Y_1}^{X_2})^2 - C$, and the profit of firm 2 is $\pi_2 = p_{X_2Y_2}Q_{X_2Y_2} + p_{X_2}Q_{X_2Y_1} + p_{Y_2}Q_{X_1Y_2} + bQ_{X_2Y_1}^2 - bQ_{X_1Y_2}^2 - I_0 - (I_{Y_2}^{X_1})^2 - C$.

Maximizing the profit with respect to p_{X_i}, p_{Y_i} respectively, and solving the first-order conditions, we derive the optimal prices

[15] For example, $b = 0.01$ implies that the royalty rate equals 1% of the amount of product Y_j sold on platform X_i; that is, firm j pays firm i $0.01 * Q_{X_iY_j}$: so if $Q_{X_iY_j} = 0.4$, then for every unit Y_j sold on platform X_i, firm j pays 0.004 to firm i.

$$\begin{cases} p^T_{X_1} = \dfrac{-bI^{X_2}_{Y_1} - bI^{X_1}_{Y_2} - I^{X_2}_{Y_1}t + I^{X_1}_{Y_2}t - 3bt + 2tv_d + 9t^2}{6t}, \\ p^T_{Y_1} = \dfrac{bI^{X_2}_{Y_1} + bI^{X_1}_{Y_2} + I^{X_2}_{Y_1}t - I^{X_1}_{Y_2}t + 3bt - 2tv_d + 9t^2}{6t}, \\ p^T_{X_2} = \dfrac{-bI^{X_2}_{Y_1} - bI^{X_1}_{Y_2} + I^{X_2}_{Y_1}t - I^{X_1}_{Y_2}t - 3bt - 2tv_d + 9t^2}{6t}, \\ p^T_{Y_2} = \dfrac{bI^{X_2}_{Y_1} + bI^{X_1}_{Y_2} - I^{X_2}_{Y_1}t + I^{X_1}_{Y_2}t + 3bt + 2tv_d + 9t^2}{6t}. \end{cases} \tag{30}$$

Substituting Eq. (30) into Eq. (26), Eq. (27), Eq. (28) and Eq. (29), we derive the optimal quantities to be

$$\begin{cases} Q^T_{X_1Y_1} = \dfrac{-I^{X_2}_{Y_1} - I^{X_1}_{Y_2} + 3t}{12t}, \\ Q^T_{X_1Y_2} = \dfrac{I^{X_2}_{Y_1} + 5I^{X_1}_{Y_2} + 9t + 4v_d}{36t}, \\ Q^T_{X_2Y_1} = \dfrac{5I^{X_2}_{Y_1} + I^{X_1}_{Y_2} + 9t - 4v_d}{36t}, \\ Q^T_{X_2Y_2} = \dfrac{-I^{X_2}_{Y_1} - I^{X_1}_{Y_2} + 3t}{12t}. \end{cases} \tag{31}$$

3.4.2. Stage 1 Firm 1 maximizes π^T_1 by optimizing $I^{X_2}_{Y_1}$, and firm 2 maximizes π^T_2 by optimizing $I^{X_1}_{Y_2}$.

We derive the first-order conditions

$$\frac{\partial \pi_1}{\partial I^{X_2}_{Y_1}} = \frac{(1-54t)I^{X_2}_{Y_1} - I^{X_1}_{Y_2} - 2v_d}{27t}, \tag{32}$$

$$\frac{\partial \pi_2}{\partial I^{X_1}_{Y_2}} = \frac{(1-54t)I^{X_1}_{Y_2} - I^{X_2}_{Y_1} + 2v_d}{27t}. \tag{33}$$

Here we assume that $54t - 1 > 0$ for the maximization of the profit.[16]

Under this assumption, Eq. (32) is less than 0, which indicates that firm 1 will be worse off if it makes an incremental investment $I^{X_2}_{Y_1}$ to make its inferior product Y_1 more valuable on the inferior platform X_2. Therefore, the optimal incremental compatibility investment of firm 1 is $I^{X_2}_{Y_1} = 0$. The system differentiation among the four systems in the market is the highest when the optimal incremental compatibility investment of firm 1 is $I^{X_2}_{Y_1} = 0$. With the highest differentiation, each of the two firms is able to increase its prices and hence its profits.[17] Realizing that $I^{X_2}_{Y_1} = 0$, firm 2's incremental compatibility investment can be solved from Equation (33) to yield

$$I^{X_1}_{Y_2} = \frac{2v_d}{54t - 1}. \tag{34}$$

[16] This assumption guarantees the existence of the optimal solution.

[17] Recall the assumption that the market is fully covered.

Substituting Eq. (34) and $I_{Y_1}^{X_2}=0$ into Eq. (30) and noting that $p_{X_1Y_1}^T=p_{X_1}^T+p_{Y_1}^T; p_{X_2Y_2}^T=p_{X_2}^T+p_{Y_2}^T; p_{X_1Y_2}^T=p_{X_1}^T+p_{Y_2}^T; p_{X_2Y_1}^T=p_{X_2}^T+p_{Y_1}^T$, the optimal prices of the systems in the market are

$$\begin{cases} p_{X_1Y_1}^T=3t, \\ p_{X_1Y_2}^T=3t+\dfrac{36tv_d}{54t-1}, \\ p_{X_2Y_1}^T=3t-\dfrac{36tv_d}{54t-1}, \\ p_{X_2Y_2}^T=3t. \end{cases} \tag{35}$$

Substituting Eq. (34) and $I_{Y_1}^{X_2}=0$ into Eq. (31), the equilibrium quantities are

$$\begin{cases} Q_{X_1Y_1}^T=\dfrac{1}{4}-\dfrac{v_d}{6t(54t-1)}, \\ Q_{X_1Y_2}^T=\dfrac{1}{4}+\dfrac{v_d(36t+1)}{6t(54t-1)}, \\ Q_{X_2Y_1}^T=\dfrac{1}{4}-\dfrac{v_d(36t-1)}{6t(54t-1)}, \\ Q_{X_2Y_2}^T=\dfrac{1}{4}-\dfrac{v_d}{6t(54t-1)}. \end{cases} \tag{36}$$

Substituting Eq. (34), Eq. (35), Eq. (36) into the profit function of each firm, we obtain the optimal profit

$$\begin{cases} \pi_1^T=\dfrac{3t}{2}+\dfrac{216tv_d^2}{(54t-1)^2}-I_0-C, \\ \pi_2^T=\dfrac{3t}{2}+\dfrac{4v_d^2}{(54t-1)}-I_0-C. \end{cases} \tag{37}$$

We summarize the results under two-way compatibility in the following proposition. (See the proof in the appendix).

Proposition 3. *The equilibrium incremental investment of firm 2 to make its superior product Y_2 more valuable when it runs on the superior platform X_1, $I_{Y_2}^{X_1}$, increases in the intrinsic value difference, v_d. The equilibrium price and quantity of the system X_1Y_2, $p_{X_1Y_2}^T$ and $Q_{X_1Y_2}^T$, increase in v_d. The equilibrium incremental investment of firm 1 to make its inferior product Y_1 more valuable when it runs on the inferior platform X_2, $I_{Y_1}^{X_2}$, equals zero. The price of the system X_2Y_1, $p_{X_2Y_1}^T$, decreases in v_d. The quantity of X_2Y_1, $Q_{X_2Y_1}^T$, decreases in v_d if $36t-1>0$. The profits of firm 1 and firm 2, π_1^T and π_2^T, increase in v_d. Under two-way compatibility, firm 1, which has the superior platform, generates more profits than firm 2, which has the inferior platform.*

Under the two-way compatibility scenario, there are four systems that are offered on the market: X_1Y_1, X_1Y_2, X_2Y_1, X_2Y_2. Among them, X_1Y_2, the superior platform with the superior product, offers the highest intrinsic value to consumers. X_2Y_1, the inferior platform with the inferior product, offers the lowest intrinsic value to consumers. X_1Y_1 and X_2Y_2 offer consumers the same intrinsic value, which is higher than the intrinsic value of X_2Y_1 and lower than that of X_1Y_2. The intrinsic value difference among these systems, system differentiation, depends on two factors: the intrinsic value difference between the superior platform/product and the inferior platform/product, v_d, and the incremental investment of firm j to add intrinsic value to product Y_j when it runs on platform X_i, $I_{Y_j}^{X_i}$.

If the intrinsic value difference v_d increases, we find that the incremental investment of firm 2 to make its superior product Y_2 more valuable on the superior platform X_1, $I_{Y_2}^{X_1}$, also increases. This indicates that firm 2 has a greater incentive to further increase the system differentiation when the intrinsic value difference increases. By increasing the incremental investment, system X_1Y_2 offers higher intrinsic value to consumers than before. This increases not only the price of the system X_1Y_2, $p_{X_1Y_2}^T$, but also the quantity of X_1Y_2, $Q_{X_1Y_2}^T$.

In contrast, the price of system X_2Y_1 decreases in the intrinsic value difference v_d, and the quantity of system X_2Y_1 decreases in v_d if $36t - 1 > 0$. When unit transportation cost t is high enough such that $36t - 1 > 0$, consumers find it more costly to switch among the systems in the market. When v_d increases, the gross utility from the system X_2Y_1, $v_{X_2Y_1}$ decreases, and because switching is more costly, the quantity demanded of system X_2Y_1 decreases, despite the lower prices. In other words, the increased transportation cost has a greater effect on the quantity of system X_2Y_1, than the lower price.

We proceed to investigate the impact of intrinsic value difference v_d on profits. Since $I_{Y_1}^{X_2} = 0$, the system differentiation depends on the intrinsic value difference v_d and the incremental investment $I_{Y_2}^{X_1}$. Notice that $I_{Y_2}^{X_1}$ also increases in v_d, and thus the larger the intrinsic value difference v_d, the higher the system differentiation. Higher system differentiation gives each firm more space to raise prices, and thereby generate more profits.

Moreover, it is worth noting that firm 1, the firm with the superior platform, generates higher profits than firm 2, the firm with the inferior platform. Noticing that $Q_{X_1Y_2}^T > Q_{X_2Y_1}^T$, the reason that firm 1 is more profitable than firm 2 lies in the royalty amount that firm 1 charges firm 2, $b(Q_{X_1Y_2}^T)^2$, which is higher than the royalty that firm 2 charges firm 1, $b(Q_{X_2Y_1}^T)^2$.

4. Comparison of different strategies

4.1. Comparison of the profit-maximizing incremental compatibility investment

Since the incremental compatibility investment only occurs under the compatibility cases, we compare the incremental compatibility investment under one-way compatibility with the superior product sold on both platforms, see Eq. (15), one-way compatibility with the inferior product sold on both platforms, see Eq. (19), and two-way compatibility, see Eq. (34).

We now summarize the incremental compatibility investment under one-way compatibility and two-way compatibility in the following proposition.[18]

Proposition 4. *When the existence conditions for each of the scenarios hold, then for one-way compatibility with the superior product sold on both platforms (OS scenario), firm 2's incremental compatibility investment increases in the intrinsic value difference between the superior platform/product and the inferior platform/product,* v_d*. In contrast, for one-way compatibility with the inferior product sold on both platforms (OI scenario), firm 1's incremental compatibility investment decreases in* v_d*. In the two-way compatible case (T scenario), firm 1 does not incrementally invest in product* Y_1*, while firm 2's incremental compatibility investment in product* Y_2 *increases in* v_d*.*

From the above comparison of incremental investments under different compatibility scenarios, we observe that firm 2 has an incentive to increase the incremental investment in the superior product Y_2 under both the OS scenario and the T scenario, as the intrinsic value difference among platform/products increases. In other words, the greater the intrinsic value difference, the more profitable it is for firm 2 to increase its incremental investment to make its superior product Y_2 more valuable on platform X_1. When the unit transportation cost t is high enough, the existence conditions for Scenario OS and T hold. In these circumstances, switching among systems in the market is costly to consumers. In these cases, firm 2 has an incentive to increase its incremental investment in making Y_2 more valuable on platform X_1, since it will increase differentiation among systems and demand for the system X_1Y_2 is going to increase, despite the higher price for that system.

We also observe that firm 1's incremental investment in making its inferior product Y_1 more valuable on the inferior platform X_2 decreases in the intrinsic value difference under the OI scenario. The greater the intrinsic value difference v_d, the lower the profit-maximizing incremental compatibility investment by firm 1 in its inferior product Y_1 and, therefore, the less valuable is the system X_2Y_1 , namely the inferior product on the inferior platform. Under the T scenario, it is optimal for firm 1 not to incrementally invest in its inferior product Y_1. This maximizes system differentiation, which in turn enables each of the two firms to raise prices and thereby increases profits.

To summarize, as the intrinsic value difference v_d increases, firm 2, which has the superior product Y_2, incrementally invests more in its product to make the system X_1Y_2 more valuable. In contrast, firm 1, which has the inferior product Y_1, has a lower incentive to increase its incremental investment in its inferior product Y_1 under the OI scenario, and chooses not to incrementally invest in its inferior product Y_1 under the T scenario.

[18] Since the existence conditions for each scenario differ, we are unable to compare the size of incremental compatibility investment across scenarios.

4.2. Comparison of profit-maximizing prices

Recall the profit-maximizing prices under non-compatibility, see Eq. (5), one-way compatibility with the superior product sold on both platforms, see Eq. (16), one-way compatibility with the inferior product sold on both platforms, see Eq. (20), and two-way compatibility, see Eq. (35).

Comparing these results to the prices of the two original systems X_1Y_1, X_2Y_2 that are in the market in each of the four cases that have been examined, and paying attention to how the prices of the compatible system are influenced by the intrinsic value difference, we can derive the following proposition:

PROPOSITION 5. *Compatibility raises the prices of system X_1Y_1 and X_2Y_2. The prices of X_1Y_1 and X_2Y_2 under two-way compatibility are the highest: $p^T_{X_1Y_1} = p^T_{X_2Y_2} > p^{\mathrm{OS}}_{X_1Y_1} = p^{\mathrm{OS}}_{X_2Y_2} = p^{\mathrm{OI}}_{X_1Y_1} = p^{\mathrm{OI}}_{X_2Y_2} > p^N_{X_1Y_1} = p^N_{X_2Y_2}$. Moreover, the higher the intrinsic value difference v_d, the higher the price of X_1Y_2, namely the system in which the superior product runs on the superior platform. In contrast, the higher the intrinsic value difference v_d, the lower the price of X_2Y_1, namely the system in which the inferior product runs on the inferior platform.*

Although each firm does not invest more in X_1Y_1, X_2Y_2 under any compatible case, the prices of these systems are higher. The reason lies in that compatibility provides consumers more choices of systems. In this way, the differentiation between each firm is greater and the competition becomes less fierce. Therefore, each firm can charge consumers a higher price compared to the non-compatible case.

Moreover, we observe that the higher the intrinsic value difference, v_d, between the superior platform/product and the inferior platform/product, the higher the price of the system X_1Y_2 under the OS Scenario and the T Scenario. In contrast, the higher the intrinsic value difference, the lower the price of the system X_2Y_1 under the OI Scenario and the T Scenario. Recall that we previously established that the higher the intrinsic value difference, the greater the incremental investment in the superior system, X_1Y_2, which in turn makes it more valuable and therefore enables a higher price for that system. In contrast, the higher the intrinsic value difference, the less incremental investment in the inferior system, X_2Y_1, which adversely affects its intrinsic value and therefore lowers the prices of the system X_2Y_1.

4.3. Comparison of equilibrium quantities

Recall the equilibrium quantities under non-compatibility, see Eq. (6), one-way compatibility with the superior product sold on both platforms, see Eq. (17), one-way compatibility with the inferior product sold on both platforms, see Eq. (21), and two-way compatibility, see Eq. (36).

Below we summarize the characteristics of quantities under different scenarios in the following proposition. (See proof in the appendix.)

Proposition 6. *The higher the intrinsic value difference v_d, the greater the quantity of X_1Y_2, the system in which the superior product runs on the superior platform. In contrast, the higher the intrinsic value difference v_d, the lower the quantity of X_2Y_1, the system in which the inferior product runs on the inferior platform under OI scenario. The relationship also holds under T scenario when 36t-1>0.*

The reason that underlies the different implications of the intrinsic value difference on profit-maximizing quantities relates to the optimal incremental investments when v_d changes: If the intrinsic value difference increases, the value of the system X_1Y_2 increases, which in turn increases its quantity. In contrast, if the intrinsic value difference increases, the value of the system X_2Y_1 decreases, which in turn decreases its quantity in the OI scenario and, when $36t-1>0$ in the T scenario.

4.4. Comparison of equilibrium profits

Recall the equilibrium profits under non-compatibility, see Eq. (7), one-way compatibility with the superior product sold on both platforms, see Eq. (18), one-way compatibility with the inferior product sold on both platforms, see Eq. (22), and two-way compatibility, see Eq. (37).

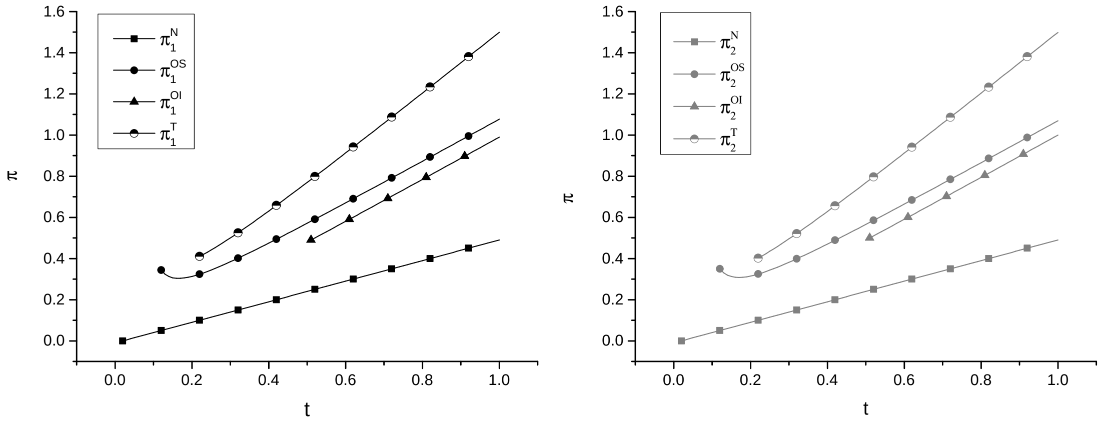
To enable a numerical comparison of profits under different scenarios, we develop a simulation.[19]

- **The relationship between the transportation cost t and profits π**

Let $b=0.1, v_d=0.5$, setting $I_0=0.01, C=0.01$, and choosing those values of $0<t<1$ that satisfy all the existence conditions of each scenario, we derive the profits under different scenarios. As depicted in Figure 3, the scenario that dominates depends on the value of the unit transportation cost, t. Recall that in the context of a digital platform, unit transportation cost can be thought of as the unit cost of adopting to a new system. When t is very small, the existence conditions of the compatibility scenarios are not satisfied; the non-compatible scenario (Scenario 1) is therefore the dominant scenario in very low ranges of t. The logic behind this result is that when t is very low, consumers' adoption costs are low and switching between systems is not costly to consumers. Competition between the two firms is intense, which adversely affect profits. Hence, the profits of each firm are too low to cover the basic compatibility investment I_0. As the transportation cost t increases, consumers' adoption costs increase, which reduces the intensity of price competition and thereby increases profits. With higher profits, each of the two firms begins to consider investing in compatibility to introduce a new system into the market, through which both firms can increase the intrinsic value difference among the systems in the market and further reduce the intensity of price competition. Recall that two systems (X_1Y_1 and X_2Y_2) with the same intrinsic value exist in the

[19] We simulated the sensitivity of these results to changes in the parameters b, v_d, and observed that the impact on the above relationship is minor. Details are available from the authors upon request.

market when neither firm wishes to invest in making its product compatible with the other firm. When t increases, we note that it becomes feasible and desirable for firm 2 to make its superior product compatible with the superior platform (OS scenario), but it is not economical for firm 1 to make any compatibility investment in that range of t. The economic rationale here is that the incremental profit of selling its inferior product Y_1 on the inferior platform X_2 is less than the basic compatibility investment plus the royalty paid by firm 1 (the superior platform X_1) to firm 2 (the inferior platform X_2), which is $(bQ_{X_2Y_1} * Q_{X_2Y_1})$. In contrast, it is optimal for firm 2 to make an incremental investment $I_{Y_2}^{X_1}$ to make its superior product Y_2 compatible on the superior platform X_1. A higher incremental compatibility investment by firm 2 in product Y_2 adds a higher intrinsic value to the system X_1Y_2. As t further increases, two-way compatibility becomes optimal with the caveat that in this scenario firm 1 only invests the basic amount I_0 to make its inferior product compatible with the inferior platform, but chooses not to make any incremental investment. Two-way compatibility increases system differentiation and thereby enables firm 1 and firm 2 to raise the prices of the systems in the market.

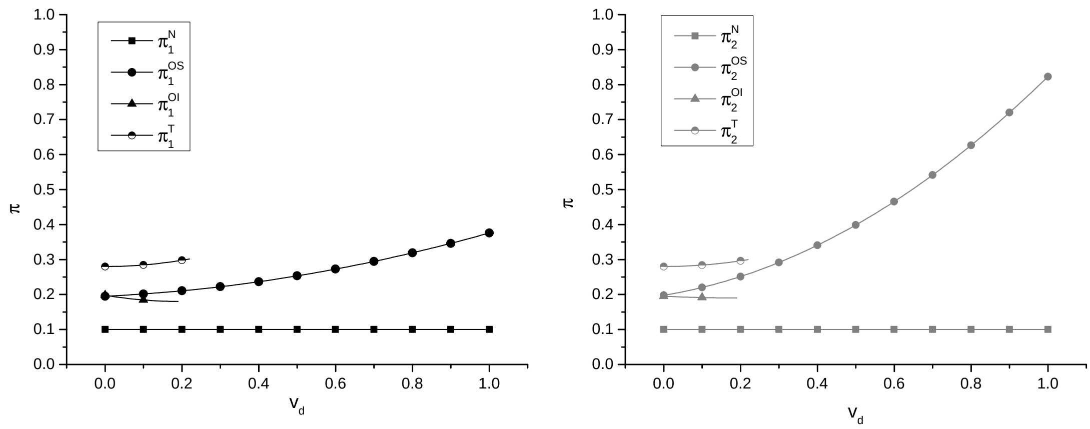


(a) Profits of firm 1 (b) Profits of firm 2

Figure 3 **The profits of these two firms under different scenarios:** $b=0.1, v_d=0.5, I_0=0.01, C=0.01$

- **The relationship between the intrinsic value difference v_d and profits π**

Let $t=0.2, b=0.4$, setting $I_0=0.01, C=0.01$, and choosing those values of $0<v_d<1$ that satisfy all the existence conditions of each scenario, we derive the profits under different scenarios (Figure 4). Given these values of t, b, I_0, C, at least one of the compatible scenarios always exists, and hence non-compatibility never dominates. Moreover, which compatible scenario dominates depends on the intrinsic value difference, v_d.



(a) Profits of firm 1 (b) Profits of firm 2

Figure 4 The profits of these two firms under different scenarios:$t = 0.2, b = 0.4, I_0 = 0.01, C = 0.01$

When v_d is small, two-way compatibility dominates. Each of the two firms chooses to list its products on both platforms. With a small intrinsic value difference, v_d, which indicates a similarity between the superior and the inferior platforms and products, both firms choose to fully mix and match in order to increase the system differentiation in the market. By doing so, they also reduce the adoption cost for consumers because consumers do not have to adopt a new platform in order to purchase its product. Greater system differentiation and mitigated adoption cost increase profitability, which in turn makes it desirable for each of the two firms to invest in making its product compatible with the platform of its rival and pay the royalty charged by the rival platform.

When v_d is large, there are two forces at work: First, the intrinsic value of the inferior system X_2Y_1 decreases, which in turn decreases the price of the system. Yet despite the lower prices, the quantity of the system also decreases when v_d is large if $36t - 1 > 0$. As a result, the profit that firm 1 derives from selling its inferior product Y_1 on the inferior platform X_2 is too low to cover the compatibility investment and the royalty paid to firm 2. Hence firm 1 will choose not to make its inferior product Y_1 available on the inferior platform X_2. Second, when v_d is large, the intrinsic value of the superior system X_1Y_2 increases and it becomes more appealing than other systems that are offered on the market, namely X_1Y_1 and X_2Y_2. The enhanced intrinsic value of system X_1Y_2 increases the price and quantity of the system X_1Y_2 simultaneously and thus increases the profits of firm 1 and firm 2. Therefore, one-way compatibility with the superior product sold on both platforms dominates if v_d is large.

5. Conclusion

An increasing number of digital platforms compete and cooperate at the same time. One way in which cooperation between platforms is manifested is through the compatibility strategies of each of the platforms. This study examines the profit-maximizing compatibility strategies in a duopoly setting of asymmetric platforms (a superior and an inferior platform) and asymmetric application products (a superior and an inferior product) by developing and analyzing a two stage game-theoretic model. We investigate the optimal prices, quantities, and the optimal incremental compatibility investment for four different compatibility scenarios: non-compatible, one-way compatibility with the superior product sold on both the superior platform and the inferior platform, one-way compatibility with the inferior product sold on both platforms, and two-way compatibility. For each scenario, we solve endogenously for the profit-maximizing incremental compatibility investment, price, and quantity, and thereby obtain the profit for each of the two firms. We proceed to compare the differences between the profits of each of the firms under different scenarios and derive the profit-maximizing compatibility strategy for these two firms under different market conditions (i.e., different parameters in the model).

Our research on the compatibility strategy of asymmetric digital platforms yields a number of insights. First, we provide informative guidance to asymmetric digital platforms on making the optimal compatibility choice. When making this choice, each digital platform firm should consider both consumers' adoption costs, and the intrinsic value difference between the superior and the inferior platforms and products. Specifically, when consumers' adoption costs are small, which implies consumers easily switch between the systems, price competition is intense and hence the profits are low. Non-compatibility is the optimal choice in this case. When adoption costs increase and/or the intrinsic value difference increases, one-way compatibility in which the superior product is made available on the superior platform is the most profitable compatibility strategy. When the adoption costs further increase, the two-way compatibility strategy is profit maximizing; namely, each of the two platform firms is incentivized to fully mix and match. It is worth noting that one-way compatibility combining the inferior product with the inferior platform is never a desirable choice for either platform firm.

Second, our research provides digital platform firms with detailed guidance on how much to incrementally invest in compatibility after a platform decides on the compatibility choice. While many dominant digital platforms choose to be compatible with their competitors, they need to decide on the size of the incremental compatibility investment. Our research addresses this issue by observing that the larger the intrinsic value difference between platforms/products, the larger should be the incremental compatibility investment when the superior application product is made compatible with the superior platform. Greater incremental compatibility investment further increases the system differentiation, and thereby benefits each digital platform firm.

With regard to the limitations of our research, we made some simplifying assumptions. First, we assume that the market is fully covered, which means that we assumed that each consumer purchases an existing system. On one hand, this assumption indicates that no matter how high the price of one system is, there are some consumers purchasing it. This effect gives platforms more choices and incentives to raise the price. On the other hand, this assumption fails to consider that market expansion may arise from compatibility. Second, for simplicity, we assume that the intrinsic value differences between the platforms and the products are the same, which may not necessarily hold in practice.

While there are simplifying assumptions that had to be made in order to be able to analytically solve the profit-maximizing compatibility strategy, prices, quantities, and the size of incremental compatibility investment, this paper contributes to the scholarly literature by: (i) filling a theoretical research void on compatibility strategies of asymmetric platforms with revenue-generating asymmetric application products; and (ii) incorporating incremental compatibility investment as an endogenous firm decision variable into the analytical model, which provides the link between the incremental compatibility investment and consumers' utility, and which has not thus far been considered in the received literature. Moreover, our analysis provides useful guidance to managers on a complex strategic issue they face.

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Appendix A: Proof of propositions

Proof of Proposition 1.

$$\frac{\partial I_{Y_2}^{X_1}}{\partial v_d} = \frac{b+3t}{81t^2-3t-b} > 0; \frac{\partial p_{X_1Y_2}^{OS}}{\partial v_d} = \frac{54t^2}{81t^2-3t-b} > 0; \frac{\partial Q_{X_1Y_2}^{OS}}{\partial v_d} = \frac{9t}{81t^2-3t-b} > 0;$$
$$\frac{\partial \pi_1^{OS}}{\partial v_d} = \frac{162t^2(t+v_d)(3t-b)}{(81t^2-3t-b)^2} > 0; \frac{\partial \pi_2^{OS}}{\partial v_d} = \frac{2(t+v_d)(b+3t)}{(81t^2-3t-b)} > 0. \quad \text{(EC.1)}$$

□

Proof of Proposition 2.

$$\frac{\partial I_{Y_1}^{X_2}}{\partial v_d} = \frac{-(3t+b)}{81t^2-3t-b} < 0; \frac{\partial p_{X_2Y_1}^{\text{OI}}}{\partial v_d} = \frac{-54t^2}{81t^2-3t-b} < 0; \frac{\partial Q_{X_2Y_1}^{\text{OI}}}{\partial v_d} = \frac{-9t}{81t^2-3t-b} < 0.$$
$$\frac{\partial \pi_1^{\text{OI}}}{\partial v_d} = \frac{-2(t-v_d)(b+3t)}{81t^2-3t-b} < 0; \frac{\partial \pi_2^{\text{OI}}}{\partial v_d} = \frac{-162t^2(t-v_d)(3t-b)}{81t^2-3t-b} < 0.$$

□

Proof of Proposition 3.

$$\frac{\partial p_{X_1Y_2}^{T}}{\partial v_d} = \frac{36t}{54t-1} > 0; \frac{\partial Q_{X_1Y_2}^{T}}{\partial v_d} = \frac{36t+1}{6t(54t-1)} > 0; \frac{\partial p_{X_2Y_1}^{T}}{\partial v_d} = -\frac{36t}{54t-1} < 0;$$
$$\frac{\partial Q_{X_2Y_1}^{T}}{\partial v_d} = -\frac{36t-1}{6t(54t-1)} < 0 \quad if \quad 36t-1 > 0;$$
$$\frac{\partial \pi_1^{T}}{\partial v_d} = \frac{432tv_d}{(54t-1)^2} > 0; \quad \frac{\partial \pi_2^{T}}{\partial v_d} = -\frac{8v_d}{54t-1} > 0. \quad \text{(EC.2)}$$
$$\pi_1^T - \pi_2^T = \frac{4(54t-1)v_d^2}{(54t-1)^2} > 0.$$

□

Proof of Proposition 4. (1) One-way compatibility with the superior product sold on both platforms:

$$\frac{\partial I_{Y_2}^{X_1}}{\partial v_d} = \frac{b+3t}{81t^2-3t-b} > 0; \quad \text{(EC.3)}$$

(2) One-way compatibility with the inferior product sold both platforms:

$$\frac{\partial I_{Y_1}^{X_2}}{\partial v_d} = -\frac{b+3t}{81t^2-3t-b} < 0; \quad \text{(EC.4)}$$

(3) Two-way compatibility case:

$$\frac{\partial I_{Y_2}^{X_1}}{\partial v_d} = \frac{2}{54t-1} > 0. \quad \text{(EC.5)}$$

□

Proof of Proposition 5. Since

$$p_{X_1Y_1}^T = p_{X_2Y_2}^T = 3t; p_{X_1Y_1}^{\text{OS}} = p_{X_2Y_2}^{\text{OS}} = 2t; p_{X_1Y_1}^{\text{OI}} = p_{X_2Y_2}^{\text{OI}} = 2t; p_{X_1Y_1}^N = p_{X_2Y_2}^N = t,$$

$p_{X_1Y_1}^T = p_{X_2Y_2}^T > p_{X_1Y_1}^{\text{OS}} = p_{X_2Y_2}^{\text{OS}} = p_{X_1Y_1}^{\text{OI}} = p_{X_2Y_2}^{\text{OI}} > p_{X_1Y_1}^N = p_{X_2Y_2}^N$ holds. Since $\frac{\partial p_{X_1Y_2}^{\text{OS}}}{\partial v_d} = \frac{54t^2}{81t^2-3t-b} > 0$; $\frac{\partial p_{X_1Y_2}^{T}}{\partial v_d} = \frac{36t}{54t-1} > 0$, the higher the intrinsic value difference v_d, the higher the price of X_1Y_2, the superior product with the superior platform. Since

$$\frac{\partial p_{X_2Y_1}^{\text{OI}}}{\partial v_d} = \frac{-54t^2}{81t^2-3t-b} < 0; \quad \frac{\partial p_{X_2Y_1}^{T}}{\partial v_d} = -\frac{36t}{54t-1} < 0;$$

the higher the intrinsic value difference v_d, the lower the price of X_2Y_1, the inferior product with the inferior platform. Hence, the proposition holds. □

Proof of Proposition 6. Since $\frac{\partial Q^{\mathrm{OS}}_{X_1Y_2}}{\partial v_d}=\frac{9t}{81t^2-3t-b}>0$; $\frac{\partial Q^{T}_{X_1Y_2}}{\partial v_d}=\frac{36t+1}{6t(54t-1)}>0$; the higher the intrinsic value difference v_d, the greater the quantity of X_1Y_2 under both the OS and T scenarios. Since $\frac{\partial Q^{\mathrm{OI}}_{X_2Y_1}}{\partial v_d}=\frac{-9t}{81t^2-3t-b}<0$; $\frac{\partial Q^{T}_{X_2Y_1}}{\partial v_d}=-\frac{36t-1}{6t(54t-1)}<0 \quad if \quad 36t-1>0$; the higher the intrinsic value difference v_d, the lower the quantity of X_2Y_1, under the OI scenario, and when *36t-1>0* under the T scenario. Hence, the proposition holds. □

Appendix B: Scenario 3: One-way compatible case: the inferior product sold on both platforms

In this scenario, the inferior product Y_1 is sold on both platforms and the superior product Y_2 is only sold on platform X_2, and the systems existing in the market are X_1Y_1, X_2Y_2, X_2Y_1. The game consists of two stages. In the first stage, firm 1 decides the incremental compatibility investment in product Y_1; in the second stage, firm 1 and firm 2 decide independently on the price of X_1, Y_1, X_2, Y_2, respectively. We use backward induction to solve the problem.

B.0.1. Stage 2 In this case, there are three systems in the market, X_1Y_1, X_2Y_2, X_2Y_1. The intrinsic value of X_1Y_1, X_2Y_2 , and X_2Y_1 are $v_{X_1Y_1}$, $v_{X_2Y_2}$, and $v_{X_2Y_1}$, respectively.

When consumers compare X_1Y_1 and X_2Y_2, the net utility of the consumer choosing X_1Y_1 is $v_{X_1Y_1}-p_{X_1Y_1}-td^{X_2Y_2}_{X_1Y_1}$, the net utility of the consumer choosing X_2Y_2 is $v_{X_2Y_2}-p_{X_2Y_2}-t(1-d^{X_2Y_2}_{X_1Y_1})$. Letting $v_{X_1Y_1}-p_{X_1Y_1}-td^{X_2Y_2}_{X_1Y_1}=v_{X_2Y_2}-p_{X_2Y_2}-t(1-d^{X_2Y_2}_{X_1Y_1})$, we can derive the location of the indifferent consumers, over which all the consumers are indifferent in purchasing X_1Y_1, X_2Y_2, then we have

$$d^{X_2Y_2}_{X_1Y_1}=\frac{t+v_{X_1Y_1}-v_{X_2Y_2}+p_{X_2Y_2}-p_{X_1Y_1}}{2t}. \tag{EC.6}$$

When consumers compare systems X_1Y_1 and X_2Y_1, the net utility for the consumer choosing X_1Y_1 is $v_{X_1Y_1}-p_{X_1Y_1}-td^{X_2Y_1}_{X_1Y_1}$, and the net utility for the consumer choosing X_2Y_1 is $v_{X_2Y_1}-p_{X_2Y_1}-t(1-d^{X_2Y_1}_{X_1Y_1})$. Similarly, we have the location of all the indifferent consumers choosing X_1Y_1 and X_2Y_1 by equating $v_{X_1Y_1}-p_{X_1Y_1}-td^{X_2Y_1}_{X_1Y_1}=v_{X_2Y_1}-p_{X_2Y_1}-t(1-d^{X_2Y_1}_{X_1Y_1})$:

$$d^{X_2Y_1}_{X_1Y_1}=\frac{t+v_{X_1Y_1}-v_{X_2Y_1}+p_{X_2Y_1}-p_{X_1Y_1}}{2t}. \tag{EC.7}$$

Consistent with the Spokes model, we have the following expression for the quantity of X_1Y_1:

$$Q_{X_1Y_1}=\frac{2(d^{X_2Y_2}_{X_1Y_1}+d^{X_2Y_1}_{X_1Y_1})}{3*2}=\frac{2t+2v_{X_1Y_1}-v_{X_2Y_1}-v_{X_2Y_2}-2p_{X_1Y_1}+p_{X_2Y_1}+p_{X_2Y_2}}{6t}, \tag{EC.8}$$

Similarly, we can obtain the following quantities of X_2Y_1 and X_2Y_2.

$$Q_{X_2Y_1}=\frac{2t+2v_{X_2Y_1}-v_{X_1Y_1}-v_{X_2Y_2}-2p_{X_2Y_1}+p_{X_1Y_1}+p_{X_2Y_2}}{6t}, \tag{EC.9}$$

$$Q_{X_2Y_2}=\frac{2t+2v_{X_2Y_2}-v_{X_1Y_1}-v_{X_2Y_1}-2p_{X_2Y_2}+p_{X_1Y_1}+p_{X_2Y_1}}{6t}. \tag{EC.10}$$

As in Scenario 2, we assume that the royalty rate is the product of constant $b\in(0,1)$ and the quantity of the new system $Q_{X_2Y_1}$. In other words, firm 2 charges firm 1 $bQ_{X_2Y_1}$ for each unit product Y_1 sold through platform X_2. In addition to the basic fixed compatibility investment I_0, the incremental compatibility investment of firm 2 , $I_{Y_1}^{X_2}$, is $(I_{Y_1}^{X_2})^2$. Therefore, the profit of firm 1 is $\pi_1^{\text{OI}} = p_{X_1Y_1}Q_{X_1Y_1} + p_{Y_1}Q_{X_2Y_1} - bQ_{X_2Y_1}^2 - I_0 - (I_{Y_1}^{X_2})^2 - C$, and the profit of firm 2 is $\pi_2^{\text{OI}} = p_{X_2Y_2}Q_{X_2Y_2} + p_{X_2}Q_{X_2Y_1} + bQ_{X_2Y_1}^2 - C$. In order to optimize each firm's profit, we assume $b < 3t$. Maximizing the profit with respect to p_{X_i}, p_{Y_i} respectively, and solving the first-order condition, we have the optimal prices

$$\begin{cases} p_{X_1}^{OI} = \dfrac{-2bI_{Y_1}^{X_2} - 3tI_{Y_1}^{X_2} - 2bt + 2bv_d + 3tv_d + 6t^2}{9t}, \\ p_{Y_1}^{OI} = \dfrac{2bI_{Y_1}^{X_2} + 3tI_{Y_1}^{X_2} + 2bt - 2bv_d - 3tv_d + 12t^2}{9t}, \\ p_{X_2}^{OI} = \dfrac{-2bI_{Y_1}^{X_2} + 3tI_{Y_1}^{X_2} - 2bt + 2bv_d - 3tv_d + 12t^2}{9t}, \\ p_{Y_2}^{OI} = \dfrac{2bI_{Y_1}^{X_2} - 3tI_{Y_1}^{X_2} + 2bt - 2bv_d + 3tv_d + 6t^2}{9t}. \end{cases} \tag{EC.11}$$

Substituting the optimal prices Eq. (EC.11) to Eq. (EC.8), Eq. (EC.9), Eq. (EC.10), we obtain the optimal quantities of systems

$$\begin{cases} Q_{X_1Y_1}^{OI} = \dfrac{8t - I_{Y_1}^{X_2} + v_d}{18t}, \\ Q_{X_2Y_1}^{OI} = \dfrac{I_{Y_1}^{X_2} + t - v_d}{9t}, \\ Q_{X_2Y_2}^{OI} = \dfrac{8t - I_{Y_1}^{X_2} + v_d}{18t}. \end{cases} \tag{EC.12}$$

B.0.2. Stage 1 Firm 1 maximizes π_1^{OI} by optimizing $I_{Y_1}^{X_2}$, the incremental compatibility investment, which further customizes the inferior product Y_1 on the inferior platform X_2. Here we assume $81t^2 - 3t - b > 0$.[20] The following optimal incremental compatibility investment is obtained by solving the first-order conditions

$$I_{Y_1}^{X_2} = \frac{bt - bv_d - 3tv_d + 3t^2}{81t^2 - 3t - b} = \frac{(3t + b)(t - v_d)}{81t^2 - 3t - b}. \tag{EC.13}$$

Substituting the Eq. (EC.13) into Eq. (EC.11) and noting that $p_{X_1Y_1}^{OI} = p_{X_1}^{OI} + p_{Y_1}^{OI}$; $p_{X_2Y_1}^{OI} = p_{X_2}^{OI} + p_{Y_1}^{OI}$; $p_{X_2Y_2}^{OI} = p_{X_2}^{OI} + p_{Y_2}^{OI}$, we derive the optimal prices of the systems in the market

$$\begin{cases} p_{X_1Y_1}^{OI} = 2t, \\ p_{X_2Y_1}^{OI} = 2t + \dfrac{54t^2(t - v_d)}{81t^2 - 3t - b}, \\ p_{X_2Y_2}^{OI} = 2t. \end{cases} \tag{EC.14}$$

Substituting Eq. (EC.13) into Eq. (EC.12), the equilibrium quantities are

$$\begin{cases} Q_{X_1Y_1}^{OI} = \dfrac{1}{2} - \dfrac{9t(t - v_d)}{2(81t^2 - 3t - b)}, \\ Q_{X_2Y_1}^{OI} = \dfrac{9t(t - v_d)}{81t^2 - 3t - b}, \\ Q_{X_2Y_2}^{OI} = \dfrac{1}{2} - \dfrac{9t(t - v_d)}{2(81t^2 - 3t - b)}. \end{cases} \tag{EC.15}$$

[20] This assumption guarantees the existence of the optimal solution.

To make sure the quantity of system $Q^{OI}_{X_2Y_1}$ is positive, we make the assumption $t > v_d$. Substituting Eq. (EC.14), Eq. (EC.15), Eq. (EC.13) into the profit function of each firm, we derive the optimal profit of each firm

$$\begin{cases} \pi_1^{OI} = t + \dfrac{(b+3t)(t-v_d)^2}{81t^2-3t-b} - I_0 - C, \\ \pi_2^{OI} = t + \dfrac{81t^2(t-v_d)^2(3t-b)}{(81t^2-3t-b)^2} - C. \end{cases} \tag{EC.16}$$