

# Efficient Bargaining Through a Broker

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## Abstract

This paper considers the possibility of efficient trade in bilateral bargaining through an informed broker. I propose a cross-subsidization mechanism that implements efficient trade in dominant strategies. I provide a condition on the broker's information such that efficient trade can be achieved. (JEL Code: C70, D82, G23)

Keywords: Asymmetric Information; Informed Broker; Cross-Subsidization Mechanism.

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# 1 Introduction

Asymmetric information can lead to market failure. In particular, the seminal work of Myerson and Satterthwaite (1983) shows that regardless of the bargaining game between a buyer and a seller, there is always a positive probability of inefficient outcomes under very mild conditions. Real world negotiations often occur through third-party brokers, like realtors in real estate transactions and investment banks in mergers and acquisitions. Moreover, brokers are likely to be informed about their market. A realtor, experienced in the real estate industry, is well informed about the market condition and the intrinsic value of a property, which are likely correlated with the seller's and buyer's private valuations. A financial intermediary is informed about the supply and demand of financial assets, and therefore knows more about the reservation values of the seller and buyer.

In this paper, I study a bargaining problem in which a broker acts as an intermediary in the bargaining between a buyer and a seller in the presence of private information. I examine the possibility of efficient trade with an *imperfectly* informed broker.<sup>1</sup> If the broker is uninformed, the problem reduces to Myerson and Satterthwaite (1983): Efficient trade is impossible. If the broker is perfectly informed, an efficient outcome can be achieved.<sup>2</sup> Intuitively, bargaining through a more informed broker can achieve higher expected gains from trade. The question I focus is whether full efficiency can be achieved with an imperfectly informed broker.

The main result of this paper is that an efficient outcome can be achieved with a broker who has *imperfect* information. I propose a class of *cross-subsidization mechanisms*, in which the broker provides the budget-imbalancing subsidy to achieve efficient trade while generating for himself a non-negative payoff. Importantly, the cross-subsidization mechanisms implement efficient trade in dominant strategies. I provide a condition on the broker's information such that efficient trade can be achieved.

The general intuition that an informed broker can improve efficiency is as follows. The pres-

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<sup>1</sup>A broker is imperfectly informed, in the sense that he observes imperfect signals about the buyer's and seller's valuations.

<sup>2</sup>A perfectly informed broker can buy from the seller at her cost and resell to the buyer at his valuation. Trade is efficient, and the broker receives all of the surplus.

ence of private information implies that the buyer and seller may misreport their valuations. In Myerson and Satterthwaite (1983), any mechanism cannot distinguish a buyer who has a valuation  $v$  and reports  $\hat{v}$  from another buyer who has a valuation  $v'$  but also reports  $\hat{v}$ . In contrast, consider bargaining through an informed broker. Since the broker's information depends on whether the buyer's valuation is  $v$  or  $v'$ , he can treat the same reports from different types of buyers differently. The broker's correlated information can help detect misreporting behaviors, therefore making it easier to induce truthful revelation.

The idea of using correlated information in mechanism design was introduced by Crémer and McLean (1985).<sup>3</sup> In an auction environment, they propose a full surplus extraction mechanism and show that a seller can extract all of the surpluses under certain conditions on the correlated information structure across buyers' valuations. Crémer and McLean (1988) consider a full surplus extraction problem in a general setting with correlated information when type spaces are discrete. McAfee and Reny (1992) extend the result to the continuous type case and show that almost full surplus extraction can be obtained. In a bilateral bargaining environment, McAfee and Reny (1992) show that an efficient budget-balanced mechanism can exist when the buyer's private valuation is correlated with the seller's.<sup>4</sup> There are two main differences in my paper. First, the cross-subsidization mechanism implements efficient trade in dominant strategies while the full surplus extraction mechanism focuses on Bayesian implementation. In this sense, the cross-subsidization mechanism is more robust than the full surplus extraction mechanism. Second, I show that full efficiency can be achieved with a partition structure, which fails to satisfy the McAfee and Reny's condition.

Brokers often play multiple roles in facilitating the exchange of a good. For instance, in the sale of a house, the realtor helps in the search process by matching a seller and a buyer; at the same time, the realtor also intermediates negotiations between the two parties. A large literature has demonstrated that brokers can alleviate search frictions, therefore improving welfare (see Ru-

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<sup>3</sup>Myerson (1981) has an example that a full surplus extraction mechanism can be achieved with correlated types.

<sup>4</sup>In the discrete type case, Gresik (1991) derives a condition for efficient trade to be feasible when private valuations are correlated.

binstein and Wolinsky (1987) among many others). My paper does not consider the search channel and focuses instead on how an informed broker can help in the bargaining.

Relatedly, Glode and Opp (2016) study the efficiency of trade in an incomplete information setting in which an uninformed seller deals with a privately informed buyer. They show that introducing informed intermediaries in a sequential trading game can expand the set of distributions of the buyer's and seller's private valuations under which efficient trade is possible.<sup>5</sup> The major difference between my paper and their paper is that, in showing that an efficient outcome can be achieved, I consider *all* mechanisms, rather than a specific game in which the asset owner makes a take-it-or-leave-it offer.<sup>6</sup> In their setting the intermediary may end up with the asset, but in my setting he optimally never retains the asset. In this sense, the intermediary in Glode and Opp (2016) acts as a dealer, whereas he acts as a broker in my setting. My paper can therefore address the applications such as how, without the possibility of holding the asset, a realtor improves trade efficiency between a buyer and a seller or how investment banks facilitate mergers and acquisitions.

Several other papers have shown that an intermediary's information can improve efficiency. Among them, Biglaiser (1993) demonstrates that the trade efficiency can be improved by the involvement of perfectly informed intermediaries. In my paper, the broker does not have perfect information. Lizzeri (1999) shows a monopolistic certification intermediary, who reveals only whether the quality is above some minimal standard, can improve efficiency. Unlike his model, the informed broker in my setup is not a certifier, but he helps by intermediating the negotiation. In a one-sided incomplete information setup, Bond and Gresik (2011) show that trade efficiency can be improved if an informed seller delegates bargaining to an intermediary. In their model, the seller can commit to outcomes negotiated by the intermediary, but in my model I do not assume the buyer and seller have such a commitment device. Biglaiser and Li (2017) study how an intermediary's information may improve efficiency by reducing the seller's moral hazard problem, which is

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<sup>5</sup>In subsequent work, Glode, Opp, and Zhang (2019) extend the analysis by showing that generically, if efficient trade can be implemented by an incentive-compatible mechanism in direct bilateral trading, it can also be achieved in a sequential trading game with a sufficiently long chain of heterogeneously informed brokers.

<sup>6</sup>In Glode and Opp (2016) and Glode, Opp, and Zhang (2019), one consequence of take-it-or-leave-it offers is that inefficiencies are partially driven by market power, which is absent in my setting.

absent in my model.

## 2 Illustrative Example

In this section, I provide a simple example to understand the main result in the paper. A seller owns an object that a buyer is interested in. The seller has a cost  $c$  of supplying the object, while the buyer values it at  $v$ . Both the cost  $c$  and the value  $v$  are private information and are drawn independently from the uniform distribution over the  $[0, 1]$  interval. The efficient allocation of the object is for the parties to trade whenever  $v > c$ . The total expected gains from trade in the efficient allocation is  $1/6$ .<sup>7</sup>

A seminal result in the bargaining literature is the Myerson and Satterthwaite (MS) theorem. The MS theorem shows that there are no budget balanced bargaining mechanisms that implement efficient trade. That is, there are always inefficiencies, regardless of the particular bargaining game that is played between the parties. For example, consider the bargaining game in which the buyer makes a take-it-or-leave-it offer to the seller. The buyer would never offer his true value  $v$ , because if he does, he will have to pay  $v$  whenever there is a transaction, being left with a surplus of 0. If, instead, the buyer shades his bid—offering  $v/2$  for example—he receives a positive expected surplus. This means that the buyer always bids less than his true value, so that trade does not happen, in some cases when  $v > c$ .

A key assumption in MS's analysis is budget balance: that the mechanism does not lose money on average. There are mechanisms that lose money but guarantee that trade is efficient. For example, consider the Vickrey-Clarke-Groves mechanism (VCG). The buyer and seller announce  $v$  and  $c$ . There is trade if  $v > c$ . In that case, the buyer pays  $c$ , and the seller receives  $v$ . This mechanism is efficient, because there is trade whenever  $v > c$ . Moreover, both the buyer and the

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<sup>7</sup>If the object is allocated efficiently, the gains from trade are  $v - c$  if  $v > c$ , and 0 otherwise. Thus, the expected surplus is

$$\int_0^1 \int_0^1 (v - c) 1_{v > c} dv dc = \frac{1}{6},$$

where  $1_{v > c}$  is the indicator function and is equal to 1 if  $v > c$ , and 0 otherwise.

seller have incentives to declare their true valuations, because they cannot affect the payments that they make or receive. However, this mechanism is not budget balanced. The mechanism designer loses the price difference  $v - c$  whenever there is trade, so that the average deficit of the mechanism equals the total gains from trade,  $1/6$ .<sup>8</sup>

The main contribution of this paper is to show that an informed broker can facilitate efficient trade while running a surplus. To see this, suppose the transaction is mediated by a broker. The broker knows which of the following three sub-intervals  $v$  belongs to:  $[0, 1/3)$ ,  $[1/3, 2/3)$ ,  $[2/3, 1]$ . Similarly, he knows which of the three sub-intervals the seller's valuation  $c$  belongs to. As shown in Figure 1, the broker knows which of the nine regions the valuations pair belongs to. I show that efficient trade can be sustained by the following mechanism. The broker first announces which region is true, then the buyer and seller play one of the following three mechanisms: fixed price mechanism, VCG, or no trade. In the top left region, that is, the seller has a valuation lower than  $1/3$  and the buyer has a valuation higher than  $2/3$ , the broker buys from the seller at a price of  $1/3$  and resells it to the buyer at a price of  $2/3$ . Trade always occurs and the broker earns a profit of  $1/3$ . In the bottom left region, the VCG mechanism is used; in this case, efficient trade can be achieved and the broker provides a subsidy. The required subsidy is  $1/18$ , because the problem is a scaled version of the original MS problem.<sup>9</sup> Similarly, in the other two diagonal regions, efficient trade is implemented and the broker provides a subsidy of  $1/18$  in each region. In the other two regions above the diagonal line, efficient trade is implemented at a fixed price of  $1/3$  and  $2/3$ , respectively. Lastly, in all of the other three regions, no trade is efficient. Overall, an efficient outcome is achieved, and the broker's ex ante expected profit is positive.<sup>10</sup> This example shows that an informed broker can use cross subsidization to implement efficient trade. I therefore refer

<sup>8</sup> Williams (1999) provides a nice proof of MS theorem by showing that the VCG mechanism always runs a deficit, and any efficient mechanism loses at least as much money as VCG.

<sup>9</sup>The required subsidy to achieve an efficient outcome is

$$\int_0^{1/3} \int_0^{1/3} (v - c) 1_{v > c} \cdot 3 \cdot 3 \, dv \, dc = \frac{1}{6} \cdot \frac{1}{3} = \frac{1}{18},$$

where  $1_{v > c}$  is the indicator function and is equal to 1 if  $v > c$ , and 0 otherwise.

<sup>10</sup>The broker's ex ante expected profit is  $(\frac{1}{3} - 3 \cdot \frac{1}{18}) \cdot \frac{1}{9} = \frac{1}{54}$ .

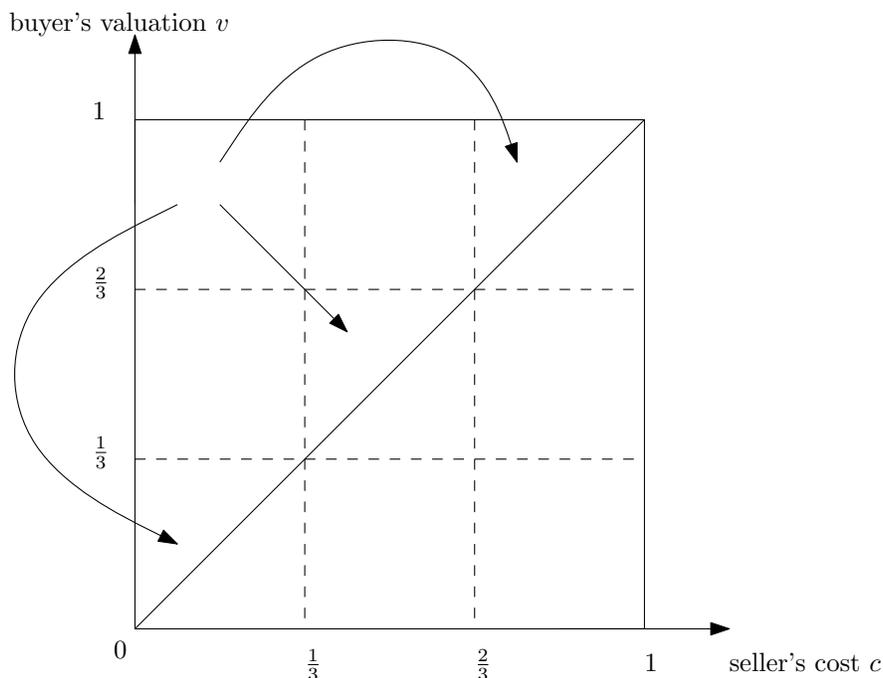


Figure 1: This figure illustrates that efficient trade can be achieved with an informed broker, when both the buyer's and seller's private valuations are drawn independently from the uniform distribution over the  $[0, 1]$  interval, and the broker knows which of the following three sub-intervals that the buyer's and seller's valuations belong to:  $[0, 1/3)$ ,  $[1/3, 2/3)$ ,  $[2/3, 1]$ . I show that the broker may use profits from the top left region to subsidize trade in the diagonal regions to implement efficient trade.

to this mechanism as a *cross-subsidization mechanism*.

In order to get the full efficiency result, the broker must be sufficiently informed. Returning to the example, suppose the broker only knows that the buyer's and seller's valuations belong to either  $[0, 1/2)$  or  $[1/2, 1]$ . In this case, the broker cannot earn profits in the top left region to subsidize trade in the diagonal regions, so efficient trade cannot be achieved by a cross-subsidization mechanism.<sup>11</sup> When the broker's information is given by a partition structure, I provide a necessary and sufficient condition on the broker's information such that an efficient outcome can be achieved. Moreover, if the broker's information divides the interval into fine enough partitions, efficient trade can be achieved.

<sup>11</sup>In the top left region, the buyer's valuation is in  $[1/2, 1]$  and the seller's valuation is in  $[0, 1/2)$ . The broker can implement efficient trade by setting a price of  $\frac{1}{2}$ , but he earns a zero profit.

## 3 Model

### 3.1 Environment

Consider a trading problem between one seller (she) and one buyer (he). The seller holds an indivisible object that is valuable for both her and the buyer. The trading is mediated by a broker, who does not enjoy consumption from the object directly. All individuals are risk neutral. Both the buyer and the seller have additively separable utility for money and the object. The broker only derives utility from money.

Let  $v$  and  $c$  denote the value of the object to the buyer and the seller, respectively. I assume that these two valuations are independent random variables. The buyer and seller know his/her own valuation with certainty, but other agents know the valuation only probabilistically. The buyer's valuation,  $v$ , is represented by a probability density function  $f$ , which is positive in the range  $[\underline{v}, \bar{v}]$ . The corresponding cumulative density function is  $F$ . The seller's cost,  $c$ , is represented by a probability density function  $g$ , which is positive in the range  $[\underline{c}, \bar{c}]$ . The corresponding cumulative density function is  $G$ . I assume that  $\underline{v} \geq 0, \underline{c} \geq 0$ . For the result to be interesting, I also assume that  $[\underline{v}, \bar{v}]$  and  $[\underline{c}, \bar{c}]$  have a non-empty intersection, in which case direct bilateral bargaining is necessarily inefficient, as in Myerson and Satterthwaite (1983).

The broker does not know the buyer's or seller's exact private valuations, but he may have some information about them. For example, a realtor, experienced in the real estate industry, is informed about the intrinsic value of a property, which is likely correlated with the seller's private valuation. A financial intermediary is informed about the supply (demand) of some financial asset, and therefore can partially assess the reservation value of a seller (buyer). Hence, I assume that the broker receives informative signals about each agent's valuation. Prior to trading, suppose the broker *privately* receives a signal  $b \in B$  of the buyer's valuation and a signal  $s \in S$  of the seller's valuation. I assume that  $(v, b)$  is independent of  $(c, s)$ . The broker's signals can be expressed in terms of conditional cumulative distribution functions,  $F^I(b|v)$  and  $G^I(s|c)$ . Let  $f^I(b|v)$  and  $g^I(s|c)$  denote the associated conditional probability density functions. The distributions ( $F$  and

$G$ ) and the conditional distributions ( $F^I$  and  $G^I$ ) are common knowledge.

In this paper, I primarily focus on a particular type of information structure that the broker may have: partition structure.<sup>12</sup> Suppose the broker knows which of the following intervals  $v$  belongs to:

$$[v_0, v_1), [v_1, v_2), \dots, [v_{n-1}, v_n],$$

and which of the following intervals  $c$  belongs to:

$$[c_0, c_1], (c_1, c_2], \dots, (c_{m-1}, c_m],$$

where  $\underline{v} = v_0 < v_1 < \dots < v_n = \bar{v}$ ,  $\underline{c} = c_0 < c_1 < \dots < c_m = \bar{c}$ , and  $m, n \geq 1$ . The set of all possible signals can be expressed as  $B = \{1, 2, \dots, n\}$  and  $S = \{1, 2, \dots, m\}$ . If  $v \in [v_{k-1}, v_k)$ , then the broker's signal is  $b = k, \forall k = 1, 2, \dots, n$ . If  $c \in (c_{k-1}, c_k]$ , then the broker's signal is  $s = k, \forall k = 1, 2, \dots, m$ . If  $m = n = 1$ , then the broker is uninformed. Denote the broker's information structure by  $I$ .

The buyer and seller are going to participate in some bargaining game mediated by the broker. Rather than explicitly modeling the process, I will study a direct mechanism in which the probability of trade and payment schedules are determined as a function of the buyer's and seller's reported valuations and the broker's signals.

By the Revelation Principle (see, e.g., Myerson (1979) and Myerson (1981)), the restriction to the direct mechanism is without loss of generality: Any outcome associated with an equilibrium of some bargaining game will also be an equilibrium outcome of some revelation mechanism in which the buyer and seller truthfully report their valuations.

Let  $p(v, c, b, s)$  be the probability that the object is transferred from the seller to the buyer, if  $v$  and  $c$  are the reported valuations of the buyer and seller, and  $b$  and  $s$  are the broker's signals. I will refer to the probability of trade,  $p(v, c, b, s)$ , as an *allocation rule*. I assume no outsider (other than the buyer, the seller, and the broker) can provide funds; thus, there are only

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<sup>12</sup>In the appendix, I study non-partition information structures. I show that my result holds as long as the broker's signals strictly shrink the support of  $F$  and  $G$ .

two payment schedules: the payment from the buyer to the broker and the payment from the broker to the seller, denoted as  $t_B(v, c, b, s)$  and  $t_S(v, c, b, s)$ , respectively. The broker's payoff is  $t_0(v, c, b, s) = t_B(v, c, b, s) - t_S(v, c, b, s)$ . Thus, a direct mechanism can be represented as  $(p(v, c, b, s), t_B(v, c, b, s), t_S(v, c, b, s))$ .

Nothing in the mechanism guarantees that the broker is willing to intermediate the transaction unless the ex ante budget balanced condition holds:

$$Et_0(v, c, b, s) \geq 0. \quad (1)$$

This condition is relevant if the broker intermediates transactions repeatedly or the broker has access to risk-neutral credit markets. To simplify exposition, throughout the paper when I discuss implementability of an allocation rule, I require that the budget balanced condition (1) holds.

### 3.2 Cross-subsidization mechanism

I examine the possibility of efficient trade, which is given by

$$\bar{p}(v, c, b, s) = \begin{cases} 1 & \text{if } v > c, \\ 0 & \text{if } v < c, \end{cases} \quad (2)$$

i.e., trade occurs with probability 1 as long as the buyer's valuation is greater than the seller's valuation, regardless of the broker's signals. I will propose a cross-subsidization mechanism, and I will provide a necessary and sufficient condition on the broker's information such that the proposed cross-subsidization mechanism implements the efficient trade in dominant strategies.

In a cross-subsidization mechanism, the broker reveals the signals he receives  $(b, s)$  and chooses

the following payment schedules:

$$\bar{t}_B(v, c, b, s) = \begin{cases} \max(c, v_{b-1}) & \text{if } v \geq c, \\ 0 & \text{if } v < c, \end{cases} \quad \text{and } \bar{t}_S(v, c, b, s) = \begin{cases} \min(v, c_s) & \text{if } v \geq c, \\ 0 & \text{if } v < c. \end{cases} \quad (3)$$

Suppose trade is efficient:  $v \geq c$ . In this case, the buyer's payment is bounded below by  $v_{b-1}$ , which is the buyer's lowest possible valuation given the broker's information; the seller would receive at least  $c_s$ , which is the seller's highest possible cost given the broker's information. Furthermore, since the buyer's payment does not depend on his report and the seller's payment does not depend on her report, honest reporting weakly dominates any other strategies.

I summarize the main result in the following proposition.

**Proposition 1.** *Suppose the broker's information is given by a partition structure,  $I$ . Then, efficient trade can be implemented in dominant strategies if and only if the following condition holds.*

$$0 \leq \sum_{b=1}^n \sum_{s=1}^m \int_{v_{b-1}}^{v_b} \int_{c_{s-1}}^{c_s} 1_{v>c} \left( v - c - \frac{F(v_b) - F(v)}{f(v)} - \frac{G(c) - G(c_{s-1})}{g(c)} \right) f(v)g(c) dc dv. \quad (4)$$

*Proof.* I first show that a necessary condition for efficient trade to be Bayesian implementable is (4). Given an efficient mechanism  $(\bar{p}, t_B, t_S)$ , I calculate the information rents that the buyer and seller must retain in the interim stage. Define

$$t_B(\hat{v}; v) \equiv \int_{\underline{c}}^{\bar{c}} \int_S \int_B t_B(\hat{v}, c, b, s) dF^I(b|v) dG^I(s|c) dG(c), \quad (5)$$

$$U_B(\hat{v}; v) \equiv G(\hat{v})v - t_B(\hat{v}; v), \quad (6)$$

$$U_B(v) \equiv U_B(v; v). \quad (7)$$

In words, for a buyer who values the asset at  $v$  and reports  $\hat{v}$ , his expected payment is  $t_B(\hat{v}; v)$ , and his expected utility is  $U_B(\hat{v}; v)$ . Consider  $v, v' \in [v_{b-1}, v_b]$ . Since reporting  $v'$  gives a weakly higher payoff than reporting  $v$  for the buyer with type  $v'$ , I obtain  $U_B(v') \geq U_B(v; v') = G(v)v' - t_B(v; v')$ . Notice that in this case the buyer's expected payment only depends on his reported

type:  $t_B(\hat{v}; v) = t_B(\hat{v}; v')$ , because the broker would have the same information  $b$ . So  $U_B(v') \geq G(v)v' - t_B(v; v) = G(v)(v' - v) + U_B(v)$ . Sending  $v' \rightarrow v$  gives to  $\frac{dU_B(v)}{dv} \geq G(v)$ . For any  $v \in [v_{b-1}, v_b)$ ,  $U_B(v) = U_B(v_{b-1}) + \int_{v_{b-1}}^v U'_B(t)dt \geq \int_{v_{b-1}}^v G(t)dt$ . As a result, I obtain a lower bound on the information rents that the buyer must retain:

$$\int_{\underline{v}}^{\bar{v}} U_B(v) dF(v) = \sum_{b=1}^n \int_{v_{b-1}}^{v_b} U_B(v) dF(v) \geq \sum_{b=1}^n \int_{v_{b-1}}^{v_b} \int_{v_{b-1}}^v G(t) dt dF(v) \equiv R_B. \quad (8)$$

Exchanging the integrals leads to an equivalent expression:

$$\begin{aligned} R_B &= \sum_{b=1}^n \int_{v_{b-1}}^{v_b} G(t)(F(v_b) - F(t)) dt = \sum_{b=1}^n \int_{v_{b-1}}^{v_b} G(v)(F(v_b) - F(v)) dv \\ &= \sum_{b=1}^n \int_{v_{b-1}}^{v_b} \left( \int_{\underline{c}}^{\bar{c}} 1_{v>c} dG(c) \right) (F(v_b) - F(v)) dv \\ &= \sum_{b=1}^n \sum_{s=1}^m \int_{v_{b-1}}^{v_b} \int_{c_{s-1}}^{c_s} 1_{v>c} \frac{F(v_b) - F(v)}{f(v)} dG(c) dF(v). \end{aligned} \quad (9)$$

Similarly, I obtain a lower bound on the information rents that the seller must retain:

$$R_S \equiv \sum_{b=1}^n \sum_{s=1}^m \int_{v_{b-1}}^{v_b} \int_{c_{s-1}}^{c_s} 1_{v>c} \frac{G(c) - G(c_{s-1})}{g(c)} dG(c) dF(v). \quad (10)$$

Denote the total surpluses from an ex post efficient mechanism by

$$W = \int_{\underline{v}}^{\bar{v}} \int_{\underline{c}}^{\bar{c}} 1_{v>c} (v - c) dG(c) dF(v). \quad (11)$$

A necessary condition for the existence of an efficient mechanism is that  $W \geq R_B + R_S$ , which is exactly the condition in the proposition.

The sufficiency is immediate by showing that the broker's ex ante expected profit in the cross-subsidization mechanism  $(\bar{p}, \bar{t}_B, \bar{t}_S)$  is given by the right-hand-side of (4). In the direct mechanism

$(\bar{p}, \bar{t}_B, \bar{t}_S)$ , if  $v \in [v_{b-1}, v_b)$ , then

$$\begin{aligned} U_B(v) &= G(v)v - \int_{\underline{c}}^{\bar{c}} \int_S \int_B t_B(v, c, b, s) dF^I(b|v) dG^I(s|c) dG(c) \\ &= G(v)v - \int_{\underline{c}}^{v_{b-1}} v_{b-1} dG(c) - \int_{v_{b-1}}^v c dG(c). \end{aligned}$$

Thus,  $U_B(v_{b-1}) = 0$ , and  $U'_B(v) = G(v)$ . Since the two inequalities in the necessary part of the proof become equalities, the broker's ex ante expected profit is equal to the total surpluses less the minimal information rents retained by the buyer and seller, i.e.,  $W - R_B - R_S$ .  $\square$

Proposition 1 provides a condition under which an efficient outcome can be achieved with an informed broker. If the distributions of the buyer's and the seller's valuations are fixed, the right hand side of equation (4) only depends on the broker's information. So Proposition 1 provides a condition on the broker's information such that efficient trade can be achieved.

In direct bilateral bargaining, Myerson and Satterthwaite (1983) prove a strong negative result: that inefficiencies always occur with a positive probability in any equilibrium under any possible bargaining game. Proposition 1 says that introducing an informed broker leads to a significantly different result: An equilibrium with the efficient outcome exists in some bargaining game. I do not claim that the efficient outcome will always be obtained. For example, the mechanism that maximizes the broker's ex ante expected utility is not necessarily ex post efficient.<sup>13</sup> Even if the efficient outcome can arise as an equilibrium in a bargaining game, I do not claim that it is the only equilibrium. The result should be interpreted as follows: Under the stated condition there exists a bargaining game with the efficient outcome such that the strong negative result in Myerson and Satterthwaite (1983) breaks.

It is intuitive to interpret the expression in (4). First,  $v - \frac{F(v_b) - F(v)}{f(v)}$  is similar to the "virtual valuation" term  $v - \frac{1 - F(v)}{f(v)}$ . The difference comes from the fact that the broker can update the distribution of  $v$  and infer the buyer's valuation from the cumulative distribution function  $\hat{F}$  given by  $\hat{F}(v) = \frac{F(v) - F(v_{b-1})}{F(v_b) - F(v_{b-1})}$ , once the broker knows which interval  $v$  belongs to. The associated

<sup>13</sup>See Zhang (2017) for the detailed analysis.

probability density function is  $\hat{f}(v) = \frac{f(v)}{F(v_b) - F(v_{b-1})}$ . Now the virtual valuation term becomes

$$v - \frac{1 - \hat{F}(v)}{\hat{f}(v)} = v - \frac{F(v_b) - F(v)}{f(v)}. \quad (12)$$

Similarly,  $c + \frac{G(c) - G(c_{s-1})}{g(c)}$  is the “virtual cost” conditional on the broker knowing  $c$  belongs to  $(c_{s-1}, c_s]$ . So the right-hand side of (4) is the broker’s expected value, whenever the buyer and seller have gains from trade, of simultaneously buying from the seller and reselling to the buyer. It must be non-negative so that the broker is willing to intermediate the bargaining.

If  $m = n = 1$ , i.e., the broker is uninformed, (4) becomes

$$0 \leq \int_{\underline{v}}^{\bar{v}} \int_{\underline{c}}^{\bar{c}} 1_{v>c} \cdot f(v) \cdot g(c) \cdot \left( v - c - \frac{1 - F(v)}{f(v)} - \frac{G(c)}{g(c)} \right) dc dv,$$

which is exactly the condition in Myerson and Satterthwaite (1983).

Recall that the broker’s information partitions the types space into  $mn$  regions:

$$I = \{[v_0, v_1), \dots, [v_{n-1}, v_n]\} \times \{[c_0, c_1), \dots, [c_{m-1}, c_m]\}.$$

Suppose  $I_1$  and  $I_2$  are two information structures. It is natural to say that  $I_1$  is more *informative* than  $I_2$  if and only if for any region in  $I_2$ , there exist weakly finer regions in  $I_1$ , and there exists at least one region such that  $I_1$  has a strictly finer partition than  $I_2$ . I present a direct result of Proposition 1.

**Corollary 2.** *If efficient trade can be achieved when the broker’s information is  $I_2$ , and  $I_1$  is more informative than  $I_2$ , then efficient trade can be achieved when the broker’s information is  $I_1$ .*

*Proof.* It is straightforward to see that the broker’s profit is higher when his signals are more informative, because a more informed broker always has the option to turn more informative signals into less informative ones and behaves as if he is a less informed broker.<sup>14</sup> Directly applying Proposition 1, the condition to achieve efficient trade is less constraining when the broker’s signals are

<sup>14</sup>See, for example, Blackwell (1951, 1953).

more informative. □

Corollary 2 does not address whether efficient trade can be achieved with a sufficiently informed broker. The next result says that there always exists a fine enough partition such that an efficient outcome can be achieved with an informed broker.

**Proposition 3.** *For any distributions of the buyer's and sell's valuations such that total gains from trade are positive, there exists a partition information structure  $I$  such that the efficient trade can be implemented in dominant strategies when the broker's information is  $I$ .*

*Proof.* It is sufficient to show that (4) holds for a particular choice of  $I$ . Suppose sub-intervals are equally distributed, i.e.,  $v_{k+1} = v_k + \frac{1}{n}$ , and  $c_{k+1} = c_k + \frac{1}{m}$ . Fix an  $\epsilon > 0$ . Suppose  $n > \frac{1}{\epsilon}$ , the minimal information rent retained by the buyer is

$$\begin{aligned} R_B &= \sum_{k=1}^n \int_{v_{k-1}}^{v_k} \int_{v_{k-1}}^v G(t) dt dF(v) \leq \sum_{k=1}^n \int_{v_{k-1}}^{v_k} \int_{v_{k-1}}^v 1 dt dF(v) \\ &\leq \sum_{k=1}^n \int_{v_{k-1}}^{v_k} (v_k - v_{k-1}) dF(v) = \frac{1}{n} < \epsilon. \end{aligned} \quad (13)$$

In other words, I can choose  $n$  large enough such that the buyer's information rent is bounded above by  $\epsilon$ . Similarly, I can choose  $m$  large enough such that the seller's information rent  $R_S < \epsilon$ . Since total gains from trade are positive, I can choose  $\epsilon$  small enough such that (4) holds. □

Proposition 3 says that as long as the broker is sufficiently informed, efficient trade can be achieved. Importantly, the broker does not have to be perfectly informed.

**Example 1.** *Assume  $v \sim U[0, 1]$ ,  $c \sim U[0, 1]$ , and sub-intervals are equally distributed, i.e.,  $v_{k+1} - v_k = 1/n$  and  $c_{k+1} - c_k = 1/m$ .*

*Consider  $m = 1$  and  $n > 1$ . Then the right-hand-side of (4) is equal to  $\frac{1-3n}{12n^2}$ , which is always negative. Efficient trade can not be sustained, but the subsidy needed from the broker to reach full efficiency is approaching to 0 as  $n \rightarrow \infty$ .*

Suppose  $m = n = 2$ , that is, the broker knows which of  $[0, 1/2)$  or  $[1/2, 1]$  the buyer's and seller's valuations fall into. Then I can calculate the right-hand-side of (4), which is  $-1/24 < 0$ . So the cross-subsidization mechanism runs a deficit.

Next, I give an example in which efficient trade exists when the broker is informed. Suppose that  $m = n = 3$ . Then the right-hand-side of (4) is given by  $1/54$ . Furthermore, by Corollary 2, if the broker's information is finer than  $m = n = 3$ , efficient trade can be achieved.

## 4 Conclusion

I show that an *imperfectly* informed broker can facilitate efficient trade in bilateral transactions. I explore a simple idea that information received by a mechanism designer can help to increase budget. While I demonstrate this idea in a simple two-agent framework, it is applicable in more general environments. For example, in the provision of public goods under asymmetric information, we know that it is generally impossible to reach an ex post efficient outcome with budget balance.<sup>15</sup> If we think of government as a broker, and it is not unreasonable to assume that the government is imperfectly informed about agents' benefit from a public goods project. In this case, a similar cross-subsidization mechanism can be used to achieve the efficient outcome.<sup>16</sup> In the cross-subsidization mechanism, the government's budget is balanced by providing subsidiary and collecting taxes.

More broadly, when the mechanism designer is informed, the literature has primarily focused on the informed principal problem, in which the informed designer chooses the mechanism that maximizes his payoff.<sup>17</sup> My paper shows that maximizing ex ante expected gains with an informed broker in a bilateral transaction can generate interesting results. Exploring how maximizing efficiency in other settings with an informed broker may produce new insights. I leave these questions for future research.

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<sup>15</sup>See, for example, Laffont and Maskin (1979) and Mailath and Postlewaite (1990).

<sup>16</sup>See the appendix for the formal analysis.

<sup>17</sup>See, for example, Myerson (1983).

# Appendix: Additional Results

## Bayesian Implementation

In this appendix, I consider Bayesian implementation of efficient trade. As in the full surplus extraction literature (e.g., Crémer and McLean (1985) and McAfee and Reny (1992)), correlated information can be quite powerful in Bayesian implementation. I show that efficient trade is Bayesian implementable under generic information structures.

Since the type spaces in my setting are continuous, I assume that the signal sets  $B$  and  $S$  are compact subsets of  $\mathbb{R}$  in order to apply the theorem in McAfee and Reny (1992). For simplicity, let  $B = [\underline{v}, \bar{v}]$  and  $S = [\underline{c}, \bar{c}]$ . I also assume that both of the joint cumulative distributions,  $F^I(v, b)$  and  $G^I(c, s)$ , have a continuous probability density function. McAfee and Reny (1992) give a necessary and sufficient condition for (almost) full rent extraction, which I first describe. I then show that if their (almost) full rent extraction condition holds, then efficient trade can be Bayesian implemented.

Let  $\Delta(B)$  denote the set of probability measures on  $B$ . Recall that  $f^I(\cdot|\cdot)$  is the conditional probability density function. I say that  $f^I(\cdot|\cdot)$  satisfies the McAfee and Reny condition if, for every  $v_0$  and every  $\mu \in \Delta(B)$ ,  $\mu(\{v_0\}) \neq 1$  implies that  $f^I(\cdot|v_0) \neq \int_{\underline{v}}^{\bar{v}} f^I(\cdot|v)\mu(dv)$ . Similarly, I say that  $g^I(\cdot|\cdot)$  satisfies the McAfee and Reny condition if, for every  $s_0$  and every  $\mu \in \Delta(S)$ ,  $\mu(\{s_0\}) \neq 1$  implies that  $g^I(\cdot|s_0) \neq \int_{\underline{c}}^{\bar{c}} g^I(\cdot|s)\mu(ds)$ . I say that the broker's information structure satisfies the McAfee and Reny condition if both  $f^I$  and  $g^I$  satisfy the McAfee and Reny condition.

**Proposition 4.** *If the broker's information structure satisfies the McAfee and Reny condition, then efficient trade is Bayesian implementable.*

*Proof.* Let  $\pi(v) = \int_{\underline{v}}^v G(t)dt$ . By the full extraction condition in McAfee and Reny (1992), I know there exists an almost full extraction. In other words, if  $\forall \epsilon > 0$ , there exists  $z_1(\cdot), \dots, z_{n_B}(\cdot)$  such that

$$0 \leq \int_{\underline{v}}^v G(t)dt - \min_n \int_b z_n(b) dF^I(b|v) \leq \epsilon. \quad (14)$$

Let  $x(v, b) = z_{n^*(v)}(b)$  where  $n^*(v) = \arg \min_n \int_b z_n(b) dF^I(b|v)$ . Consider  $t_B(v, c, b, s) = c1_{v>c} + x(v, b)$ . Now

$$\begin{aligned}
U_B(\hat{v}; v) &= vG(\hat{v}) - \int_{\underline{v}}^{\hat{v}} t dG(t) - \int x(\hat{v}, b) dF^I(b|v) \\
&\leq vG(v) - \int_{\underline{v}}^v t dG(t) - \int x(\hat{v}, b) dF^I(b|v) \\
&= \int_{\underline{v}}^v G(t) dt - \int x(\hat{v}, b) dF^I(b|v) \\
&= \int_{\underline{v}}^v G(t) dt - \int z_{n^*(\hat{v})}(b) dF^I(b|v) \\
&\leq \int_{\underline{v}}^v G(t) dt - \int z_{n^*(v)}(b) dF^I(b|v) \leq \epsilon.
\end{aligned} \tag{15}$$

The first two inequalities can be achieved if  $\hat{v} = v$ . Thus, IC constraints hold. In addition,  $0 \leq U_B(v) \leq \epsilon$ . □

To understand the proposition, I explain the McAfee and Reny condition. It is easier to explain its discrete counterpart, in which case the condition becomes the Crémer and McLean (1988) condition. Basically, the Crémer and McLean condition says that the vector of conditional probabilities corresponding to any possible value is not in the convex hull of the vectors of conditional probabilities corresponding to other possible types. This condition on the information structure is a spanning condition. Roughly speaking, Crémer and McLean (1988) show that we can design a “side bet” with the following properties: It gives the buyer a large negative payoff if he does not report truthfully; it charges the truth-telling buyer the exact amount of payoff such that he is willing to participate. The spanning condition is exactly what we need to invert a system of linear equations to design the desired “side bet”.

Given the above intuition, I consider first a truth-telling mechanism in the bargaining stage (second stage) supplementing a “pre”-mechanism (first stage). In the second stage, the buyer pays the seller’s reported valuation and the seller receives the buyer’s reported valuation, if the buyer’s reported valuation is greater than the seller’s. Honesty is a dominant strategy for both the buyer and

the seller. I then supplement the following first-stage mechanism, in which the broker offers menus of participation fee schedules to extract surpluses from the buyer and seller. Specifically, for the buyer, the broker can offer a set of participation fee schedules  $\{z_n(\cdot)\}$ . Prior to the bargaining, the buyer chooses one of the participation fee schedules  $z_n(\cdot)$ , which requires the buyer to pay  $z_n(b)$  if the broker's signal is  $b$ . Since the broker's information structure satisfies the McAfee and Reny condition, I can design the participation fee schedules such that the buyer earns a rent of no greater than  $\epsilon$ ,  $\forall \epsilon > 0$ . Since the total gains from trade is positive, the mechanism is budget-balanced if I choose a sufficiently small  $\epsilon$ . Since the McAfee and Reny condition holds generically (McAfee and Reny, 1992), I conclude that efficient trade is Bayesian implementable with a generic informed broker.

Given Proposition 4, I emphasize that there are two main differences between my paper and McAfee and Reny (1992). First, the cross-subsidization mechanism implements efficient trade in dominant strategies while Proposition 4 focuses on Bayesian implementation. In this sense, the cross-subsidization mechanism is more robust than the full surplus extraction mechanism.

Second, partition structures fail to satisfy the McAfee and Reny's condition. To see this, recall from the proof of Proposition 1 that the buyer and seller retain positive information rents when the broker has a partition information structure. In contrast, under the McAfee and Reny's condition, given  $\epsilon > 0$ , there exists a mechanism so that information rents left to the buyer and seller is no greater than  $\epsilon$ .

## **Non-Partition Information Structures**

In this appendix, I generalize the cross-subsidization mechanism, and I show that it is possible to achieve efficient trade under non-partition information structures. A key ingredient is that the broker's signals strictly shrink the support of  $F$  and  $G$ . I present the following result.

**Proposition 5.** *Suppose the broker's signals satisfy the following conditions:  $b \leq v$  and  $s \geq c$ .*

Consider the payment schedules:

$$\bar{t}_B(v, c, b, s) = \begin{cases} \max(c, b) & \text{if } v > c, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and } \bar{t}_S(v, c, b, s) = \begin{cases} \min(v, s) & \text{if } v > c, \\ 0 & \text{otherwise.} \end{cases}$$

If  $\mathbb{E}(\bar{t}_B(v, c, b, s) - \bar{t}_S(v, c, v, s)) \geq 0$ , then efficient trade can be implemented in dominant strategies.

*Proof.* Under the above payment schedules, truthful reporting is a weakly dominant strategy because payments do not depend on their own report when there is trade. Second, since the broker's profit is given by  $\mathbb{E}(t_B(v, c, b, s) - t_S(v, c, v, s))$ , which is non-negative, the budget balanced condition holds. So efficient trade can be implemented in dominant strategies.  $\square$

Note that Proposition 5 requires that the realization of the signal on the buyer's valuation is weakly smaller than the true valuation and the realization of the signal on the seller's cost is weakly greater than the true cost. This guarantees that the buyer's and seller's ex post individually rational constraints hold in the direct mechanism with the above payment schedules. Indeed, a buyer whose valuation is  $v$  at most pays  $b$  and a seller whose cost is  $c$  receives at least  $s$  whenever  $v > c$ . In fact, I can relax the assumption. The essential conditions I need are that the support of the buyer's valuation conditional on the broker's signal is a shrinking version of  $[\underline{v}, \bar{v}]$  and the support of the seller's valuation conditional on the broker's signal is a shrinking version of  $[\underline{c}, \bar{c}]$ .

I use the following example to illustrate the proposition.

**Example 2.** Suppose that  $v, c \sim U[0, 1]$  and  $F^I(b|v) = \left(\frac{b}{v}\right)^\theta$  for  $b \in [0, v]$  and  $G^I(s|c) = 1 - \left(\frac{1-s}{1-c}\right)^\theta$  for  $s \in [c, 1]$ , where  $\theta \geq 0$  is a parameter that measures the informativeness of the information structures. The interpretation is as follows. If  $\theta$  is an integer, the structure can be viewed as an inference problem from  $\theta$  independent observations.<sup>18</sup> As a result, a larger  $\theta$  implies

<sup>18</sup>The broker's signal on the buyer's valuation can be viewed as the broker receives  $\theta$  observations drawn independently from the distribution  $U[0, v]$ . The broker's signal on the seller's cost can be viewed as the broker receives  $\theta$  observations drawn independently from the distribution  $U[c, 1]$ .

a more informative environment in the Blackwell's sense.<sup>19</sup> The information structure can allow for non-integer  $\theta \geq 0$ .

Consider the following direct mechanism

$$\bar{p}(v, c, b, s) = \begin{cases} 1 & \text{if } v > c, \\ 0 & \text{otherwise,} \end{cases}$$

$$\bar{t}_B(v, c, b, s) = \begin{cases} \max(c, b) & \text{if } v > c, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and } \bar{t}_S(v, c, b, s) = \begin{cases} \min(v, s) & \text{if } v > c, \\ 0 & \text{otherwise.} \end{cases}$$

The broker's payoff is  $h(\theta) \equiv \mathbb{E}(\bar{t}_B(v, c, b, s) - \bar{t}_S(v, c, v, s))$ . Integrating over  $(v, c, b, s)$ , the broker's payoff is  $h(\theta) = \frac{\theta+1}{3(\theta+2)} - \frac{\theta+4}{6(\theta+2)} = \frac{\theta-2}{6(\theta+2)}$ .

If  $\theta = 0$ , the information structure is completely uninformative. In this case,  $h(0) = -1/6 < 0$ , implying efficient trade can not be achieved by the proposed direct mechanism. It is straightforward to verify that the broker's payoff is an increasing function of  $\theta$ , implying that it is easier to sustain efficient trade if the broker is more informed.

If  $\theta \geq 2$ , then the broker's payoff is non-negative:  $h(\theta) \geq 0$ , and efficient trade can be achieved. As  $\theta \rightarrow \infty$ , the information structure approaches to the perfectly informative structure, and the broker's payoff  $\lim_{\theta \rightarrow \infty} h(\theta) = 1/6$ , which is the total gain from trade.

## Asymmetric Information Bargaining Problems with Many Agents

In this appendix, I extend the result in Proposition 1 to bargaining problems with many agents (Laffont and Maskin, 1979; Mailath and Postlewaite, 1990). The most pervasive application of such a problem is the provision of public goods. I introduce a simple and new model to illustrate the result.

There are  $n$  agents, who must decide whether to undertake a public project, and if undertaken,

<sup>19</sup>See Blackwell (1951) and Blackwell (1953).

how to distribute the costs among agents. The cost of providing the public project is  $C$ . Agent  $i$ 's valuation of the project is  $v_i$ , which is agent  $i$ 's private information. Other agents only know that  $v_i$  is drawn from a distribution function  $F_i$  with support  $[\underline{v}_i, \bar{v}_i]$ . Denote the probability density function by  $f_i$ , which is positive in the range  $[\underline{v}_i, \bar{v}_i]$ . I assume that  $\underline{v}_i \geq 0$  and  $v_i$ s are independent random variables. All agents are risk neutral, and agents have additively separable utility for the money and the project.

There is a broker who mediates the bargaining problems with the  $n$  agents. The broker's objective is to maximize the total surpluses in the economy. The broker is informed in the sense that the broker receives a signal  $s_i$  about each agent's valuation  $v_i$ . Suppose the broker knows which of the following intervals  $v_i$  belongs to:

$$[v_i^0, v_i^1), [v_i^1, v_i^2), \dots, [v_i^{k_i-1}, v_i^{k_i}],$$

where  $\underline{v}_i = v_i^0 < v_i^1 < \dots < v_i^{k_i} = \bar{v}_i$  and  $k_i \geq 1$ . The set of possible signals can be expressed as  $\{1, 2, \dots, k_i\}$ . If  $v_i \in [v_i^{k-1}, v_i^k)$  for some  $k \in \{1, 2, \dots, k_i\}$ , then the broker's signal is  $s_i = k$ .

We can focus on direct mechanisms. Let  $p(v_1, \dots, v_n, s_1, \dots, s_n)$  be the probability that the public project is undertaken. Let  $t_i(v_1, \dots, v_n, s_1, \dots, s_n)$  denote the cost contributing schedule for agent  $i$ . A mechanism is feasible if the following budget balanced condition holds:

$$Ep \left( \sum_i t_i - C \right) \geq 0.$$

An ex post efficient mechanism requires that the public project is undertaken whenever the total benefit exceeds the total cost, i.e.,

$$\bar{p}(v_1, \dots, v_n, s_1, \dots, s_n) = \begin{cases} 1 & \text{if } \sum_{j=1}^n v_j > C, \\ 0 & \text{if } \sum_{j=1}^n v_j < C. \end{cases}$$

I summarize the result in the following proposition.

**Proposition 6.** *Suppose the broker's information is given by a partition structure. Then, the broker can implement the ex post efficient outcome in dominate strategies if and only if the following condition holds.*

$$0 \leq \sum_{s_1=1}^{k_1} \cdots \sum_{s_n=1}^{k_n} \int_{v_1^{s_1-1}}^{v_1^{s_1}} \cdots \int_{v_n^{s_n-1}}^{v_n^{s_n}} 1_{\sum v_i > C} \left[ \sum_{i=1}^n \left( v_i - \frac{F_i(v_i^{s_i}) - F_i(v_i)}{f_i(v_i)} \right) - C \right] dF_1(v_1) \cdots dF_n(v_n). \quad (16)$$

*Proof.* The proof is similar to the proof of Proposition 1. I outline the proof with minimal details. First, I can show that a necessary condition for the ex post efficient outcome to be Bayesian implementable is (16). The key is to show that there is a lower bound on the information rent that agent  $i$  must retain

$$R_i = \sum_{s_1=1}^{k_1} \cdots \sum_{s_n=1}^{k_n} \int_{v_1^{s_1-1}}^{v_1^{s_1}} \cdots \int_{v_n^{s_n-1}}^{v_n^{s_n}} 1_{\sum v_i > C} \left( \frac{F_i(v_i^{s_i}) - F_i(v_i)}{f_i(v_i)} \right) dF_1(v_1) \cdots dF_n(v_n).$$

Denote the total surpluses from an ex post efficient mechanism by

$$W = \sum_{s_1=1}^{k_1} \cdots \sum_{s_n=1}^{k_n} \int_{v_1^{s_1-1}}^{v_1^{s_1}} \cdots \int_{v_n^{s_n-1}}^{v_n^{s_n}} 1_{\sum v_i > C} \left[ \sum_{i=1}^n v_i - C \right] dF_1(v_1) \cdots dF_n(v_n). \quad (17)$$

A necessary condition for the existence of an ex post efficient mechanism is that  $W \geq \sum_{i=1}^n R_i$ , which is exactly the condition in the proposition.

In order to show the sufficiency, consider the following cost contributing schedule for agent  $i, \forall i$ :

$$\bar{t}_i(v_1, \dots, v_n, s_1, \dots, s_n) = \begin{cases} \max(C - \sum_{j \neq i} v_j, v_i^{s_i-1}) & \text{if } \sum_{j=1}^n v_j > C, \\ 0 & \text{if } \sum_{j=1}^n v_j < C. \end{cases}$$

Since  $\bar{t}_i$  does not depend on  $v_i$  when  $\sum_{j=1}^n v_j > C$ , honest reporting is a weakly dominant strategy. Similar to the proof of Proposition 1, the broker's ex ante expected payoff in the direct mechanism  $(\bar{p}, \bar{t}_1, \dots, \bar{t}_n)$  is exactly given by the right-hand-side of (16). This completes the proof.  $\square$

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